JOURNAL OF
THE ENVIRONMENTAL
ENGINEERING DIVISION

Total Phosphorus Model for the Great Lakes

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Introduction

The major effort to control eutrophication of the Great Lakes has been directed towards reducing the input of phosphorus (14). Knowledge of the sources and sinks of this nutrient within the basins of each of the Great Lakes could facilitate the implementation of such a strategy; however, the size of the system makes direct measurement costly and difficult. As an alternative to direct measurement, a mathematical model is presented that simulates total phosphorus budgets for each of the Great Lakes. A feature of this model is that its waste sources are derived from variables reflecting human development in the basin. As such, it represents a tool to estimate the long-term effect of human activities on the water quality of the Great Lakes.

Since the model is intended to compute long-term trends, average annual values are simulated and variability within the year is ignored. On such a time frame, the lakes are treated as completely mixed systems with the exception of Lake Erie, which is divided into its three subbasins. With these time and space scales, the approach is as depicted in Fig. 1. A waste source model is used to simulate phosphorus input based on variables such as population and land use. These loads are then input to a phosphorus budget model that represents a mass balance around each lake. The budget model accounts for transport between lakes as well as in-lake losses and can be used to calculate in-lake concentration of total phosphorus as a function of time.

Such an approach has been successfully applied to chloride, a conservative substance (24). The primary purpose of the present paper is to demonstrate its applicability to a nonconservative substance such as phosphorus. To do this, a historical simulation is developed from census information for the period 1800–1970 and compared with data. Additionally, some tentative predictions are made to demonstrate the model's use as a planning tool.

Note.—Discussion open until September 1, 1977. To extend the closing date one month, a written request must be filed with the Editor of Technical Publications, ASCE. This paper is part of the copyrighted Journal of the Environmental Engineering Division, Proceedings of the American Society of Civil Engineers, Vol. 103, No. EE2, April, 1977. Manuscript was submitted for review for possible publication on April 30, 1976.

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Model Structure

A mass balance around a completely mixed lake results in the following differential equation:

$$\text{accumulation} = \begin{matrix}\text{waste}\\ \text{sources}\end{matrix} + \begin{matrix}\text{inflow from}\\ \text{upstream lake}\end{matrix} - \text{outflow} - \begin{matrix}\text{in-lake}\\ \text{losses}\end{matrix} \quad \ldots\ldots\ldots (1)$$

$$V\frac{dp}{dt} = W(t) + Q_u p_u - Qp - s$$

in which V = volume of the lake; p = concentration of total phosphorus; t = time; Q_u = flow from an upstream lake; p_u = concentration of the upstream lake; Q = outflow from the lake; and s = rate of in-lake losses. Concentration is the dependent variable and the flows and volumes of the Great Lakes are known (12,27). The following sections deal with the in-lake losses and waste sources for the model.

In-Lake Losses.—It is well established that a sizeable fraction of incoming

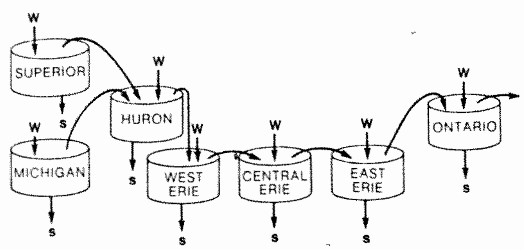

FIG. 1.—Phosphorus Budget Model of Great Lakes (W Represents Waste Sources of Total Phosphorus from Lake's Drainage Basin; s Represents In-Lake Losses to Sediments)

phosphorus is incorporated into lacustrine sediments (11). Although some release from the sediments occurs, particularly when oxygen is depleted from the hypolimnion (3,17), the net flux of phosphorus on an annual basis is into the sediments (39). One of the first attempts to account for this loss was to represent it as a monomolecular decay (37):

$$s = \sigma V p \quad \ldots\ldots\ldots (2)$$

in which σ = the first-order rate constant. Although Eq. 2 has been shown to be an adequate approximation of the long-term phosphorus dynamics of a single lake (33), it is unsuitable as a general relationship for all lakes. Vollenweider (38) used data to demonstrate that σ is not constant from lake to lake, but is inversely related to lake depth. An explanation based on theoretical considerations (5) is that Eq. 2 represents uniform loss throughout the lake volume, whereas phosphorus is lost as a transport across the sediment-water interface. Eq. 2 is a realistic description for a *homogeneous* reaction, like radioactive decay, in which all particles have an equal probability of reacting. The loss

of phosphorus, however, is a *heterogeneous* reaction, and the removal could therefore be rewritten as

$$s = \nu A_s p \quad \ldots\ldots\ldots (3)$$

in which ν = the apparent settling velocity of total phosphorus; and A_s = the surface area of the sediments (which is assumed equal to the lake's surface area).

The settling velocity is called "apparent" to indicate that it is an artifact of the model's structure and time scale. In actuality, within the yearly cycle an atom of phosphorus is subject to many transformations and a variety of transport mechanisms on its way to the sediments. Therefore, the apparent settling velocity represents a macroscopic parameter that is proposed to describe net in-lake losses of total phosphorus *on an annual basis.* By using the budget model (Eq. 1) to relate ν to measureable quantities, the apparent settling velocity can be determined empirically. First Eqs. 1 and 3 are combined and expressed as

$$V\frac{dp}{dt} = W' - Qp - \nu A_s p \quad \ldots\ldots\ldots (4)$$

in which W' is the total input of phosphorus, i.e., the inflow from upstream lakes plus waste sources. At steady state (dp/dt = 0), Eq. 4 can be solved for phosphorus concentration, yielding

$$p = \frac{W'}{Q + \nu A_s} \quad \ldots\ldots\ldots (5)$$

For a lake at steady state, phosphorus retention, R_p, is defined as the fraction of the annual input of total phosphorus which is retained by the sediments

$$R_p = \frac{\text{loss to sediments}}{\text{total input}} = \frac{\nu A_s p}{W'} \quad \ldots\ldots\ldots (6)$$

Eq. 5 can then be substituted into Eq. 6 and simplified to yield a relationship between apparent settling velocity and measureable quantities

$$R_p = \frac{\nu}{\nu + q_s} \quad \ldots\ldots\ldots (7)$$

in which q_s = the areal overflow rate = Q/A_s.

Dillon and Rigler (8) have measured R_p and q_s for a set of lakes in southern Ontario. Since these lakes lie within the Great Lakes' basin and are typical of the region (i.e., climate and geology), they represent an excellent data base for the evaluation of ν. A least squares fit to evaluate the settling velocity (5) results in a value of 52.7 ft/yr (16 m/yr) which is then used in the simulations for the Great Lakes.

Waste Source Model.—Summaries of the potential sources of total phosphorus are reported elsewhere (26,36). The present model considers the following categories. The point source is domestic waste, which includes human waste and detergents. The diffuse sources are: land runoff, which includes agricultural, urban, and forested land; and atmospheric sources.

The categorization is based on present knowledge of phosphorus inputs to the Great Lakes. For example, industrial sources are not included since it is assumed that they typically contribute minimal amounts of phosphorus (26); measurements on the Great Lakes support this assumption (29). Additionally, the causal relationships of certain sources could not be modeled explicitly due to a lack of information. These sources are included in one of the preceding categories; for instance, animal wastes and pesticides are assumed to be reflected in agricultural runoff.

Calculations are done on a lake drainage basin level which introduces a possible source of error to the loading estimates as the model makes no distinction between phosphorus that originates far from or in close proximity to the lake. Losses may occur in transit to the lake, particularly if the tributary stream flows through an impoundment where phosphorus could be trapped in the sediments. With this in mind, the calculations are as follows.

Domestic Waste.—Domestic sources are the sewered wastewater from residences, businesses, and institutions. Although domestic phosphorus may be attributable to a variety of causes, an overwhelming proportion is due to human waste and detergents (20,26). The contribution from human waste, $W_h(t)$, is calculated for each drainage basin by

$$W_h(t) = S(t)\,[1 - T(t)]\,C_h\,P(t) \quad \text{(8)}$$

in which $S(t)$ = the fraction of the population served by sewers; $T(t)$ = the fraction of the total phosphorus removed by treatment; C_h = the annual per capita generation rate of total phosphorus from human waste; and $P(t)$ = population.

The contribution due to detergents $W_d(t)$ is

$$W_d(t) = S(t)\,[1 - T(t)]\,C_d(t)\,P(t) \quad \text{(9)}$$

in which $C_d(t)$ = the annual per capita generation rate of total phosphorus due to detergents. Eqs. 8 and 9 indicate that only sewered domestic wastewater will reach the lake. This is based on the assumption that unsewered waste will filter into the soil via septic tanks and outhouses, where in most cases phosphorus is not particularly mobile (26).

Land Runoff.—Phosphorus loading from the land has been attributed to several factors, including land use, topography, precipitation, soil characteristics, vegetative cover, manipulative practices, and animal populations. On the basis of their review of published data, Uttormark, et al. (35) concluded that delineation beyond a broad characterization of land use is presently unwarranted; therefore, a classification scheme of agricultural, urban, and forested land is used. The waste load from land drainage, $W_l(t)$, can be expressed as

$$W_l(t) = A_d \sum_{j=1}^{3} F_j(t)\,L_j \quad \text{(10)}$$

in which j = 1, 2, 3, for agricultural, urban, and forested land, respectively; A_d = the drainage area of the lake; $F_j(t)$ = the fraction of the drainage basin devoted to land use j; and L_j = the phosphorus load per unit area for land use j.

Atmospheric Sources.—Atmospheric loading $W_a(t)$ is the contribution from

TABLE 1.—Summary of Parameters Used in Historical Simulation

Parameter (1)	Source (2)
Flow, Q; Volume, V; Surface area, A_s; Drainage area, A_d	(12,27)
Population, $P(t)$	U.S. Census of population; Census of Canada; (25)
Fraction of population served by sewers, $S(t)$	During the nineteenth and early twentieth centuries, assumed equal to the percentage of the population living in urban areas (U.S. Census of population; Census of Canada). Present values taken from Ref. 15.
Fraction of total P in sewered wastewater removed by treatment, $T(t)$	Assumed linear increase from zero removal in 1900 to 5% in 1940 reflective of primary treatment. Removal then increased linearly to 20% in 1970 reflective of secondary treatment. Removal efficiencies taken from Ref. 31.
Annual per capita generation rate of total P due to human waste, C_h	Value of 1.1 lb/capita/yr (0.5 kg/capita/yr) taken from Refs. 20, 34, and 36.
Annual per capita generation rate of total P due to detergents, $C_d(t)$	(10)
Annual atmospheric loading of total P per unit of lake surface area, $L_a(t)$	Values of 0.21 lb/acre/yr (24 kg/km²/yr), 0.27 lb/acre/yr (30 kg/km²/yr), and 0.32 lb/acre/yr (36 kg/km²/yr) are used for present input to Lakes Superior and Huron; Michigan; Erie and Ontario, respectively (unpublished study by Matheson and Ref. 22). Historically, 0.21 lb/acre/yr (24 kg/km²/yr) used for period when watershed was completely forested. Increases to present values occur linearly from 1850 to 1900 when agriculture and logging became prominent in the Basin.
Fraction of lake drainage area devoted to agriculture, $F_1(t)$	U.S. Census of Agriculture; Census of Canada; personal communication, Gene Jarecki (Great Lakes Basin Commission), David Gierman (Environment Canada).
Fraction of lake drainage area devoted to urban land, $F_2(t)$	Historically, assumed to be proportional to urban population. Present values from personal communication (Jarecki, Gierman).
Fraction of lake drainage area devoted to forest, $F_3(t)$	Calculated as total land minus urban and agricultural land.
Annual loading of total P per unit of drainage area devoted to land use, L_j	1.34 lb/acre/yr (150 kg/km²/yr) for urban land (35); 0.4 lb/acre/yr (5 kg/km²/yr) for forest land in Canadian Shield (7); 0.18 lb/acre/yr (20 kg/km²/yr) for all other forest (35); 0.89 lb/acre/yr (100 kg/km²/yr) for U.S. agricultural land in drainage basin of Western Lake Erie (2); 0.36 lb/acre/yr (40 kg/km²/yr) for all other agricultural land (1).

dustfall, rainfall, and snowfall, and is represented in the model as

$$W_a(t) = A_s\, L_a(t) \quad \text{. . . . . (11)}$$

in which $L_a(t)$ = the atmospheric loading rate per unit area of lake surface.

The phosphorus loading, $W(t)$, for each lake is the summation of Eqs. 8–11. The loading serves as the forcing function for the set of equations that result when Eq. 4 is written for each of the Great Lakes. An Euler-Cauchy numerical integration scheme is used to obtain solutions for these equations.

Historical Simulation (1800–1970)

Parameters.—A description of all model parameters would be too voluminous for the present publication and will be reported elsewhere (in preparation). Table 1 presents a summary.

Initial Conditions.—Phosphorus loadings to the Great Lakes prior to 1800 were due primarily to atmospheric input and forested land runoff. Eq. 10 is used to calculate the contribution from land runoff by using the forested runoff rates presented in Table 1. The present input to Lake Superior of 0.21 lb/acre/yr (24 kg/km²/yr) from atmospheric sources is assumed to have applied to all the Great Lakes prior to 1800. Initial conditions are then calculated by allowing the model to reach a steady state under these loadings.

The resulting concentrations suggest a physical limitation to water quality improvement in the basin. Due to its large drainage basin to surface area ratio (an order of magnitude greater than the other lakes), Western Lake Erie's calculated concentration is nearly 50% higher than the next highest lake. This calculation suggests that total removal of cultural wastes would never bring Western Erie to the levels possible in the other lakes.

Results.—The calculated phosphorus loadings are shown in Fig. 2. Lake Erie's loading has been aggregated since past measurements have been reported as such. In general, the lakes have been subject to two major periods of increased phosphorus load. First, during the latter part of the nineteenth century, the change from forested to agricultural land use caused increases in all the basins. Second, since about 1945, increased sewering, population growth, and the introduction of phosphate detergents made a strong impact, particularly on Lakes Erie, Ontario, and Michigan. The effect of the recent increase was smaller for Huron and Superior due to the lower population density of their watersheds. It could be expected; therefore, that treatment of point sources would have little effect on these lakes. Note that of all the lakes, Ontario is the only one significantly influenced by an upstream lake. This implies that the recovery of Lake Ontario is dependent on the state of Lake Erie, and that a coordinated program of waste abatement is necessary for the two lakes.

The calculated concentration profiles are presented in Fig. 3. As with the phosphorus loadings, several of the lakes have experienced rapid acceleration in concentration of total phosphorus over the past 20 yr. However, the magnitude of the increase is modified by the morphology of the lake. For instance, Fig. 2 shows that in 1970 Lake Michigan and Lake Ontario had similar loadings of total phosphorus. Due to the larger surface area of its sediments, however, Lake Michigan's concentration is about 50% of that of Lake Ontario's.

Since phosphorus budgets are just being measured, a historical verification

of the loadings is impossible. While present estimates are meager, they are available and a comparison with the model results is presented in Fig. 4. While it is difficult to draw conclusions from such a small sample, the fact that the *untuned* model achieves a respectable correlation coefficient for both loadings ($r = 0.89$) and concentration ($r = 0.94$) is encouraging. A more detailed comparison of the model's performance is presented in Table 2, where the calculated

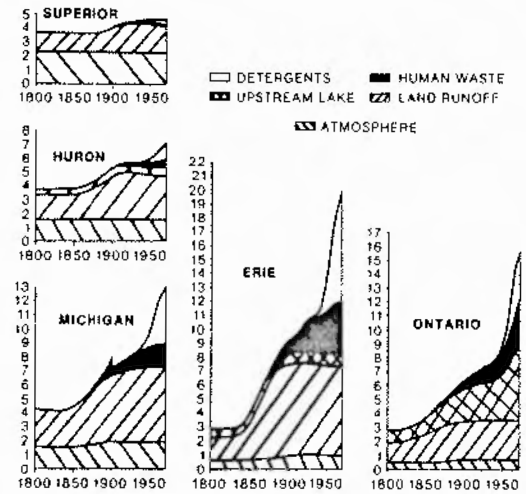

FIG. 2.—Historical Loadings of Total Phosphorus, in thousands of tons per year, as Calculated by Model

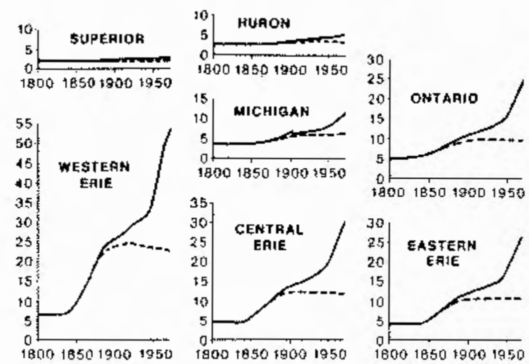

FIG. 3.—Historical Concentrations of Total Phosphorus, in micrograms per liter, as Calculated by Model (Dashed Line Represents Concentration due to Diffuse Sources)

components of the budget are compared with measurements for Lake Ontario. Again, considering that the model is untuned, the results agree reasonably well.

In general, the model overestimates concentration, particularly for Central and Eastern Lake Erie. This implies that in-lake losses to the sediments in these basins may be higher than calculated by the model. This could be due to the geology of Lake Erie's drainage basin which causes inordinately large

TABLE 2.—Comparison of Model Results for 1970 with Phosphorus Budget for Lake Ontario Measured During 1972 by Casey and Salbach (4)—All Data in Thousands of tons (Millions of kilograms)

Budget variables (1)	Casey and Salbach (2)	Model (3)
Sources:		
Niagara River	8.4 (7.6)	7.1 (6.0)
Tributaries	3.7 (3.4)	5.5 (5.0)
Atmospheric	1.9 (1.7)	0.8 (0.7)
Municipal	3.9 (3.5)	3.0 (2.7)
Industrial	0.1 (0.1)	—
Ground water	0.0 (0.0)	—
Total	18.0 (16.2)	16.4 (14.4)
Sinks:		
St. Lawrence River	8.7 (7.9)	6.3 (5.7)
Retained in lake and sediments	9.1 (8.3)	9.6 (8.7)

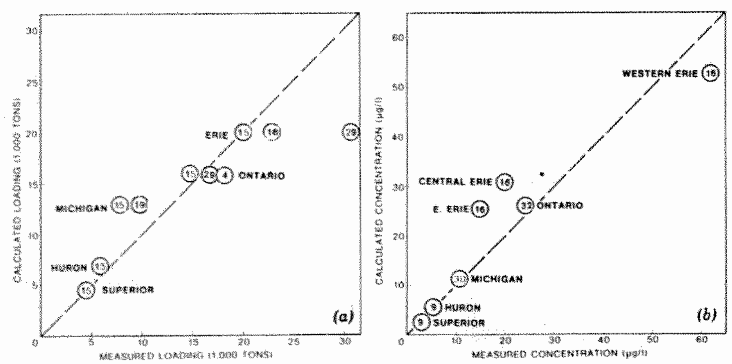

FIG. 4.—Comparison of measurements and Calculations for Present (1965–1975) Values of Loadings and Concentration with: (a) Loadings in thousands of tons per year; (b) Concentration in micrograms per liter (Encircled Numbers Refer to References from which Measured Values Are Taken)

quantities of suspended solids to enter the lake from stream runoff. A second explanation is that the assumption of complete mixing might be rendered invalid by enhanced loss of phosphorus to the sediments in the nearshore. Whether this loss would be significant at the time scale of the present model is unknown and is beyond the scope of this paper.

Future Simulation (1970–2000)

The model can now be used to make tentative predictions of the magnitude and rate of recovery of the Great Lakes under a phosphorus removal program. Although phosphorus abatement is currently underway, the status of the program is uncertain enough to discourage its direct use as a basis for projections. Instead, an idealized scenario which approximately conforms to the present goals of the program will be employed to predict the lakes' response.

Parameters.—The Great Lake states and the Province of Ontario have embarked on phosphorus reduction programs that concentrate initially on reduction of domestic sources. In general, a goal of 1 mg/l of total phosphorus in the effluent of all point sources is a good approximation to the present overall objective (13,15). The scenario for recovery therefore assumes a linear decrease in point source loading from 1970 values to a 1-mg/l effluent in 1980. The latter date approximately conforms to the established timetable. In a sense the linearity of the decrease is unrealistic since the treatment of huge point sources like Detroit would be better described as a step decrease. However, the idealization will give the average effect of such reductions.

The 1-mg/l effluent can be translated into a model variable by assuming that 150 gal/capita/day (570 l/capita/day) is typical of the volume of waste generated by an inhabitant of the basin. This number is derived from sewage plant data from the basins of Lakes Erie and Ontario (29) and is assumed to apply to the other lakes. Multiplying 150 gal/capita/day by 1 mg/l yields an equivalent per capita loading rate of 0.46 lb/capita/yr (0.21 kg/capita/yr) of total phosphorus as the treatment objective for 1980.

It is assumed that all other parameters, with the exception of human population and urban land use (which is assumed to be proportional to population), remain fixed at 1970 values. Population projections were taken from publications of the Water Resources Council (23) and by extrapolation of present trends.

Initial Conditions.—There are several possibilities for initial conditions for the future simulation. One would be to tune the historical model by adjusting the apparent settling rate so that the calculated concentrations at the present time would exactly match measured values. Such manipulation would result in higher settling velocities for Central and Eastern Erie and Ontario, which would tend to quicken the recovery of these basins. Since the simulation is to a certain extent speculative, a conservative approach is taken by using the uncalibrated model.

Results.—The response of the Great Lakes to the reduction of point source effluents to 1 mg/l is shown in Fig. 5. To aid interpretation of the results, total phosphorus concentration is related to trophic state by assuming that mesotrophy is bounded by 10 μg/l and 20 μg/l of total phosphorus (6). By using this guideline, all but the Western Basin of Lake Erie will recover rather rapidly to less than eutrophic levels by 1985. In addition, Lake Michigan will return to an oligotrophic state.

Rate of Recovery.—Fig. 5 demonstrates a rapid rate of recovery, with all lakes approaching a steady state before 1985. Since a gradual decrease in loading was used, the actual rate is somewhat obscured. If the response time is defined as the time required to reach 90% of a new steady state following a step decrease

TABLE 3.—Ninety Percent Response Times of Great Lakes Due to Reduction of Point Sources to 1-mg/l Effluent in 1970

Lake (1)	90% response time, in years (2)
Superior	20
Michigan	16
Huron	9
Western Erie	0.25
Central Erie	1.5
Eastern Erie	2.5
Ontario	7.7

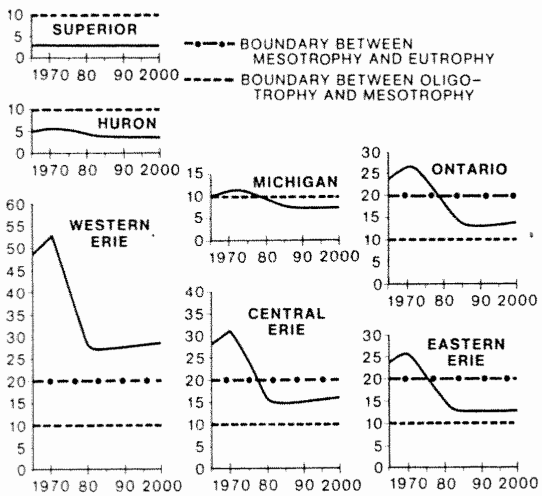

FIG. 5.—Future Simulation of Concentration of Total Phosphorus, in micrograms per liter, in Great Lakes in Response to Linear Decrease in Point Source Loadings from 1970 Values to 1-mg/l Effluent in 1980

in loading, it can be calculated using the model. Rainey (28) did this for the Great Lakes as isolated entities (no upstream influence) for conservative substances with resulting values ranging from about a decade for Lake Erie to half a millenium for Lake Superior. Interpretation of this calculation as appropriate for nonconservative substances could lead to the misconception that the lakes may take decades to centuries to recover from waste reductions. This assumption

does not apply to decaying substances since in-lake losses enhance recovery. For the future simulation, the recovery time can be calculated by running the model with 1970 initial conditions and an immediate step decrease to the 1980 loading levels. The response times (Table 3) indicate that the lake's response in terms of total phosphorus is a maximum of two decades.

There are two serious qualifiers to these predictions. The first concerns the assumption that the loss of phosphorus to the sediments is a one-way first-order reaction. Although this assumption may be reasonable for most of the Great Lakes, its validity for Lake Erie is open to question for a number of reasons:

1. In cases where phosphorus has risen to high concentrations in the sediments due to heightened waste inputs, feedback of sediment phosphorus may occur as concentration in the water column drops. This has been illustrated for Lake Washington by Lorenzen (21) and might retard the recovery of Lake Erie, which has experienced the highest loadings of all the Great Lakes.
2. If a lake's hypolimnion goes anoxic due to the decomposition of organic matter during the summer, feedback of phosphorus becomes greatly enhanced (17). Burns and Ross (3) have shown that in the Central Basin of Lake Erie, anoxia results in an 11-fold increase in the rate of release during parts of the summer. If such a condition persists after phosphorus removal programs have been implemented, recovery could be retarded further.
3. Erie's Western and Central basins are both shallow enough so that storms can reintroduce substantial quantities of phosphorus to the water column through suspension of the sediments. If this phosphorus, which is predominantly in a particulate form, stays in the water column long enough to influence the lake's productivity, it could also slow recovery.

A second qualifier to the predictions is that they may overestimate point and underestimate diffuse sources. The former is due to the model's lack of distinction between point sources that originate far from or in close proximity to the lake. The latter is due to the fact that the estimates of land runoff used in the present model were taken from studies that primarily sampled streams during low flow periods. Recent investigations (2,18) have demonstrated that sizeable portions of nutrient runoff may enter lakes during high flow-short duration episodes in the spring.

The aforementioned qualifers imply that the model's projections are on the optimistic side. This should be kept in mind when evaluating the results. All the previously mentioned phenomena are beyond the scope of the present investigation, but are being addressed in ongoing research.

Practical Applications

By design, the approach presented in this paper has been structured to make it useful in judging the effects of present policies on the future water quality of the Great Lakes. An example of such an application was presented in the preceding section. An additional feature of the approach is that the model's computer software is also designed with management decisions in mind. The computer program is structured so that political or geographic distinctions can be made when calculating model parameters from county data. Thus, it is relatively easy to develop scenarios of future conditions which ask questions such as, "What would happen if the State of Michigan outlawed detergents, while all other parts of the region did not?" or "How much of the phosphorus from domestic waste is contributed by people living in counties contiguous with one of the lakes?" Such questions give an idea of the sort of scenarios that are possible with the present model and that may be of use to decision makers.

Summary and Conclusions

A model of total phosphorus budgets for the Great Lakes has been presented. Agreement between model predictions and presently measured values of total phosphorus concentrations and loadings are consistent enough to conclude that the model gives a reasonable first estimate of total phosphorus levels on the Great Lakes. A historical simulation indicates that the lakes experienced two major increases in phosphorus since 1800. The first was due to the agriculturalization of the basin in the latter part of the nineteenth century. The second started in about 1945 and is the result of increased population and sewer construction and the introduction of phosphorus detergents. A simulation of future conditions was also presented as an example of the use of the model in decision-making. Use of the approach for decision-making will be improved by a number of refinements that are presently being incorporated into the model. These include a more mechanistic handling of diffuse sources and in-stream losses, addition of sediment-water interactions, and the explicit inclusion of nearshore areas and embayments to the model segmentation. When such modifications are made, the approach will offer a comprehensive and relatively inexpensive package for investigating man's impact on the future water quality of the Great Lakes.

Acknowledgments

Janice Boyd, Andrew Robertson, Donald Scavia, and Stephen Tarapchak of GLERL made helpful suggestions during the preparation of this manuscript. I would also like to express appreciation to Raymond Canale of the University of Michigan and William Sonzogni of the Great Lakes Basin Commission for their encouragement of this research.

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Appendix II.—Notation

The following symbols are used in this paper:

A_d = drainage area of lake;
A_s = surface area of lake;
C_d = annual per capita generation rate of total P due to detergents;
C_h = annual per capita generation rate of total P due to human waste;
F_j = fraction of lake's drainage area devoted to land use j;
L_a = annual atmospheric loading of total P per unit of lake surface area;

L_j = annual loading of total P per unit of drainage area devoted to land use j;
P = human population;
p = concentration of total phosphorus;
Q = flow;
q_s = areal overflow rate;
R_p = phosphorus retention coefficient;
S = fraction of population served by sewers;
s = rate of in-lake losses of total P to the sediments;
T = fraction of total P in sewered wastewater removed by treatment;
t = time;
V = volume of lake;
v = apparent settling velocity of total P;
W = waste sources of total P from lake's drainage basin;
W' = waste sources from drainage basin plus inputs of total P from upstream lakes;
W_a = atmospheric sources of total P;
W_d = detergent sources of total P;
W_h = human waste sources of total P;
W_l = land runoff sources of total P; and
σ = first-order rate constant for in-lake losses of total P.

Subscripts

j = 1, 2, and 3 for agricultural, urban, and forest runoff, respectively; and
u = upstream lake.