J. Great Lakes Res., 1979
Internat. Assoc. Great Lakes Res. 5(1):73-77.

RELATIVE ACCURACY OF CONNECTING CHANNEL DISCHARGE DATA WITH APPLICATION TO GREAT LAKES STUDIES[1]

Frank H. Quinn
Great Lakes Environmental Research Laboratory
Environmental Research Laboratories, NOAA
Ann Arbor, Michigan 48104

ABSTRACT. The flows in the Great Lakes connecting channels are a major component in the water balance of the Great Lakes Basin. The increased emphasis on Great Lakes water quality and quantity requires an assessment of the accuracy of both measured and computed connecting channel discharge data. In this study, the standard error of typical discharge measurements was found to be approximately 3 to 5 percent, depending upon the number of panels used in the cross section. Mesurement sets were found to have a practical limit of about 25 measurements. The standard error of a set of measurements was found to be on the order of 1 percent. The procedure used to compute the published flows of the Niagara River was found to have an apparent bias of about 2 percent on the high side. It is recommended that the published Niagara River flows be adjusted prior to use in detailed water balance studies.

INTRODUCTION

The flows in the Great Lakes connecting channels are a major component of the water balance of the Great Lakes Basin. Two of the channels, the Detroit and St. Clair Rivers, are not continually measured. Their discharges are computed by stage-fall-discharge equations (Quinn 1978) or by mathematical transient models (Quinn and Wylie 1972). On the other hand, the flows in the Niagara, St. Lawrence, and St. Marys Rivers are continually monitored by power plant ratings, compensating gate ratings, and in the case of the Niagara River, the Maid-of-the-Mist pool rating equation.

With the increased emphasis on the water quality and quantity of the Great Lakes, it is necessary to evaluate the relative accuracy of the various discharge calculations for use in Great Lakes studies. This paper addresses two important aspects of Great Lakes discharges. The first is the accuracy of the discharge measurements used to calibrate equations and models and to compute water quality loadings. The second is a comparison of the power plant ratings for the Niagara and St. Lawrence Rivers with discharge measurements taken during the International Field Year for the Great Lakes (IFYGL).

DISCHARGE MEASUREMENT PROCEDURE

Typical connecting channel discharge measurement procedures as applied to the Niagara and St. Lawrence Rivers are given by the Water Survey of Canada (1972) and Cox (1974). The Niagara River was measured at the Stella Niagara section located 2 miles below Queenston, Ontario. The measuring section, 550 m wide, was divided into 12 panels. The interior panels consisted of eight 43 m-wide panels and two 49 m-wide panels. The two end panels were 52 m wide and 50 m wide. Four Price current meters were used to measure the velocity at 0.1 depth increments in each panel. This required three different meter settings of 2 minutes each. Three current meters were adjusted to new depths after each setting in each panel, while the fourth meter was held constant at the 0.4 depth to correct for flow fluctuations. The current meters were individually rated and a separate rating curve developed for each meter.

Similar procedures were applied to the St. Lawrence River with the measuring section located at Iroquois Dam. The St. Lawrence section consisted of 15 panels rather than the 12 used on the Niagara.

[1]GLERL Contribution No. 143.

DISCHARGE MEASUREMENT ANALYSIS

The error analysis of the discharge measurements used the procedures recommended by Carter (1970) and Herschy (1970). Each of these procedures computes the standard error of a discharge measurement based upon the technique involved. Carter (1970) computes the overall standard error of a discharge measurement, XQ, from

$$XQ^2 = S^2_{R_i} + S^2_{R_t} + S^2_{R_s} + S^2_{R_N} \tag{1}$$

where S_{R_i} is the standard error due to instrument error, 1.0 percent

S_{R_t} is the standard error due to velocity pulsation

S_{R_s} is the standard error due to vertical velocity curve errors

S_{R_N} is the standard error due to the number of measuring stations in the cross section.

The velocity pulsation error is given by

$$S_{R_t} = \frac{S_{r_t}}{\sqrt{N_p}} \tag{2}$$

where S_{r_t} is the standard error due to velocity fluctuations at a point

N_p is the number of observation points

The standard error due to the point sampling of velocity in the vertical is given by

$$S_{R_s} = \frac{S_{r_s}\,[1 + (N\text{-}1)\,\rho]^{1/2}}{\sqrt{N}} \tag{3}$$

where S_{r_s} is the standard error due to the shape of the vertical velocity curve at a station

N is the number of panels in a cross section

ρ is the average correlation coefficient, 0.04, between the ratios of the average 0.3 and 0.8 depth velocities and the mean in the vertical for a given section.

Carter (1970) provides a table of values for S_{r_t}, S_{r_s}, and S_{R_N}. Applying Carter's (1970) procedure to Niagara River, we obtain

$$XQ^2 = 1.0^2 + \left(\frac{2.9}{\sqrt{12}}\right)^2 + \left(\frac{4.3\,[1 + (12\text{-}1)0.004]^{1/2}}{\sqrt{12}}\right)^2 + 4.1^2$$

$$XQ = 4.6 \text{ percent}$$

The 95 percent confidence level (2XQ) is then 9.2 percent. For the Niagara River, Carter's (1970) tables indicate

S_{r_t} is 2.9 for 2 minute measurements

S_{r_s} is $\leqslant$ 4.3 for vertical measurements at each 0.1 depth

S_{R_n} is 4.1 for 12 panels.

The 95 percent confidence level for the set of Niagara River discharge measurements, 24 in total, is given by

$$2\,\overline{XQ} = \frac{2\,XQ}{\sqrt{N_m}} = \frac{9.2}{\sqrt{24}} = 1.9 \text{ percent} \tag{4}$$

where N_m is the number of measurements.

Herschy (1970) computes the overall standard error, XQ, of a discharge measurement by

$$XQ = \pm\sqrt{(X'_q)^2 + (X''_q)^2} \tag{5}$$

where X'_q is the overall random error in discharge

X''_q is the overall systematic error in discharge.

The overall random error in discharge is given by

$$X'_q = \sqrt{X'^{\,2}_m + \frac{1}{m}(X'^{\,2}_b + X'^{\,2}_d + X'^{\,2}_v)} \tag{6}$$

where X'_m is the error due to the choice of the number of verticals

X'_b is the error in measuring width

X'_d is the error in measuring depth

X'_v is the error in measuring velocity

m is the number of panels.

The error in measuring velocity is given by

$$X'_v = \pm\sqrt{\frac{X_f^2}{P} + X_o^{\,2}} \tag{7}$$

where X_f is the error due to the choice of duration of exposure of current meter

X_o is the error due to the choice of the number of points in a vertical

P is the number of measuring points in the vertical.

The overall systematic error in discharge is

$$X_q = \pm\sqrt{X_b''^2 + X_d''^2 + X_v''^2} \quad (8)$$

where X_b'' is the systematic error in measuring width
X_d'' is the systematic error in measuring depth
X_v'' is the systematic error of the current meter.

Herschy (1970) also provides tables to aid in evaluating the above errors. By applying Herschy's procedure to the Niagara measurements, the following values are obtained from his tables

X_m' = 3.6 percent for 12 panels
X_f' = 6 percent
X_o' = 0.5 percent for using the 9 point velocity distribution
X_b' is recommended as 0.5
X_d' = 1.5 percent

Thus

$$X_v' = \sqrt{\left(\frac{6^2}{9} + 0.5^2\right)} = 2.06$$

$$X_q' = \left(3.8^2 + \frac{1}{12}(0.5^2 + 1.5^2 + 2.06^2)\right)^{1/2} = 3.87.$$

The systematic error is computed as follows:

X_b'' = 0.5 percent
X_d'' = 0.5 percent
X_v'' = 0.5 percent for velocity greater than 0.3 m/sec

$$X_q'' = \sqrt{(0.5^2 + 0.5^2 + 0.5^2)} = 0.87.$$

The overall standard error then becomes

$$XQ = \left(3.87^2 + 0.87^2\right)^{1/2} = 3.97 \text{ percent.}$$

The 95 percent confidence level 2XQ then becomes 7.9 percent. This is approximately 1.3 percent less at the 95 percent confidence level than computed by Carter's procedure. The difference in the results is largely due to the differing percentages ascribed to the error induced by the number of panels in the cross section. This is the largest contributor to error in both procedures. In actuality, the overall error is probably somewhat less than indicated in both procedures because transverse coefficients are determined from the measured transverse velocity curve and applied to each of the panels in the discharge measurement computations. The flow variation between the mesurements was relatively small with the coefficient of variation being approximately 7 percent. A correlation analysis between the percent error and the flow rate indicated very little correlation with the coefficient of determination being approximately 12 percent.

For the St. Lawrence River measurements the basic procedure given above for the Niagara River was followed except that 15 rather than 12 panels were used in the cross section. This results in standard errors, XQ, of 4.5 and 3.2 percent by Carter's and Herschy's procedures, respectively. The difference is due to Carter's step function for S_{R_N}. Because the step function changes rapidly between the 15th and 16th panels, there is a reduction of 1.7 percent in the standard error. Thus, Herschy's value of 3.2 percent is the more probable value. The standard error of the mean set of 19 measurements then becomes

$$XQ = \frac{XQ}{\sqrt{N_m}} = \frac{3.2}{\sqrt{19}} = 0.7 \text{ percent.}$$

The preceding analysis can also serve as a guide to achieve desired accuracy in an individual or a set of discharge measurements. Using standard connecting channel flow measuring procedure, accuracy can be improved by increasing the number of panels in the case of an individual discharge measurement and also by increasing the number of measurements in the case of a group of measurements. Figure 1 shows the variation in the standard error of a single discharge measurement with the number of panels in the cross section. The impact of Carter's step function for S_{R_N}, as mentioned earlier, is readily apparent. It is also noted that a reduced rate of improvement results from using over 25 panels. Figure 2 shows the standard error of a set of measurements versus the number of measurements for the situation where the error of a single discharge measurement is 9.2 percent at the 95 percent confidence level. It is seen that the optimum number of measurements in a set would approach 25. Above 25 measurements the incremental increase in accuracy for additional measurements is substantially reduced.

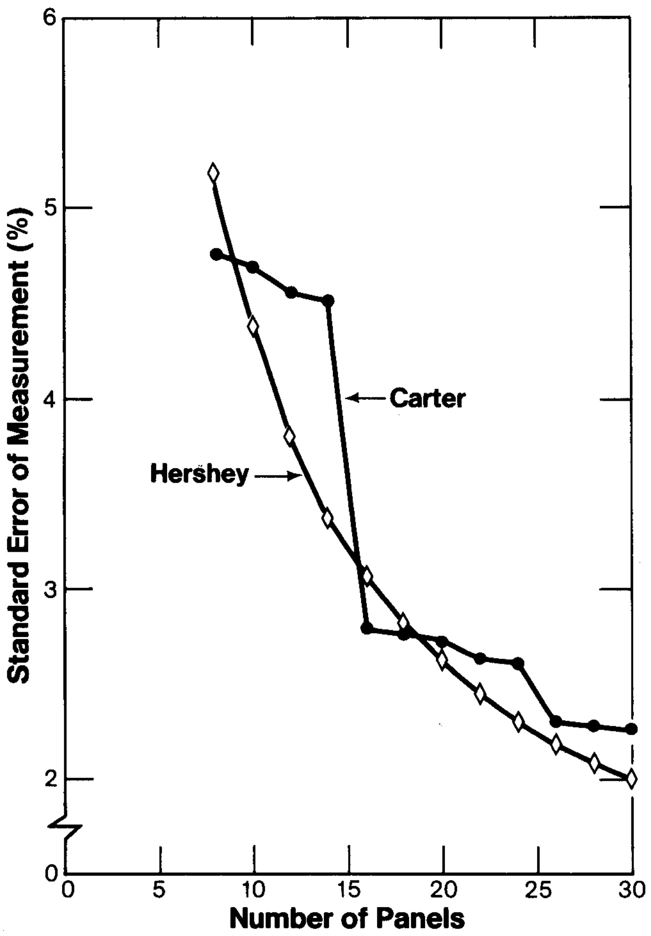

FIG. 1. Standard error of measurement vs. number of panels in the cross-section.

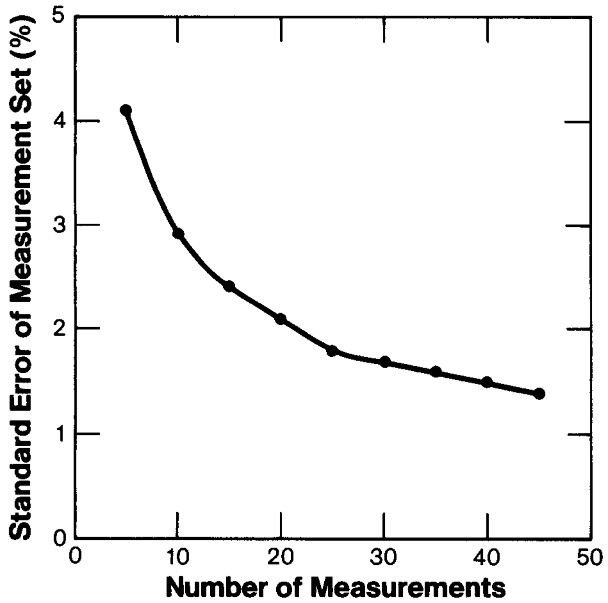

FIG. 2. Standard error of measurement set vs. number of measurements for XQ = 9.2% at the 95% confidence limit.

COMPARISON OF DISCHARGE MEASUREMENTS AND POWER PLANT RATINGS

The aforementioned discharge measurements were taken during IFYGL to verify the power plant ragings used to compute the Niagara and St. Lawrence River flows on an operational basis. If either of the ratings were biased, corrections should be applied prior to use in Lake Ontario water balance studies. The plant ratings and measured discharges were compared using the paired Student's t test. Table 1 gives the differences between the plant ratings and the measured Niagara River discharges, expressed as a percentage of the measured flow (Water Survey of Canada 1972).

The Student's t is given by

$$t = \frac{\mu_p}{S_p} N_m^{\frac{1}{2}}$$

where μ_p is the mean percentage difference between the paired measured and rating flows

S_p is the standard deviation of the percent differences

N_m is the number of measurements.

The Student's t for the 24 Niagara River measurements is given by

$$t = \frac{1.9\,(24)^{\frac{1}{2}}}{1.05} = 8.86$$

At the 1 percent significance level, 0.5 percent in each tail, the critical value of t for 23 degrees freedom is 2.81. Since this is less than the t value of 8.86, the power rating appears to be biased. Thus, the recorded Niagara River discharges should be corrected by 1.9 percent prior to use in sensitive Lake Ontario water balance studies. Also, additional measurements should probably be taken to verify the above conclusion.

The impact of a 1.9 percent change in Niagara River inflow on water balance studies is that it reduces computed water balance evaporation by approximately 200 mm/year or about 30 percent.

The differences between the power plant rating and measured St. Lawrence River discharges for the 19 measurements are given in Table 2. The Student's t for the St. Lawerence River is

$$t = \frac{0.49\,(19)^{\frac{1}{2}}}{1.08} = 1.98$$

As the computed value of 1.98 is less than the

TABLE 1. Comparison of measured and power plant ratings of Niagara River flows at Stella at the Niagara Section, 1972.

Measurement Number	Percent Difference	Measurement Number	Percent Difference
1	0.7	22	2.0
3	1.6	23	3.2
4	1.0	24	2.7
5	1.1	25	1.3
7	0.8	26	0.9
9	2.3	27	2.8
10	0.5	28	1.9
12	1.8	29	1.0
14	2.5	30	3.3
15	2.0	31	3.4
16	0.2		
17	3.5	Mean μ_p	1.9
18	3.5	Std. Dev. S_p	1.05
19	0.9		

TABLE 2. Comparison of measured and power plant ratings of St. Lawrence River flows at Iroquois Dam, July 1972.

Measurement Date	Percent Difference	Measurement Date	Percent Difference
6	2.9	20	-0.8
7	0.3	21	-0.1
8	1.1	22	1.0
11	0.6	24	0.1
12	1.2	25	-0.8
13	0.4	26	1.6
14	0.2	27	-0.9
15	1.6	28	-0.1
17	0.1		
18	2.1	Mean μ_p	0.5
19	-1.1	Std. Dev. S_p	1.08

critical value of 2.88 for 18 degrees of freedom at the 1 percent significance level, the St. Lawrence power ratings appear unbiased and recorded values can be used without correction in the Lake Ontario studies.

CONCLUSIONS

The standard error of individual connecting channel discharge measurements is found to be on the order of 3 to 5 percent, depending upon both the procedure used and the number of panels in the cross section. The optimal number of panels from an error standpoint appears to be approximately 25. Figure 1 can be used to assess the errors involved when practical considerations limit the number of panels.

Measurement sets used in the calibration of discharge equations or models should comprise approximately 25 measurements. Above this number a relatively small incremental gain in accuracy is obtained for an increased number of measurements. Figures 1 and 2 can be used to design a set of measurements to meet varying objectives.

Of particular importance to Lake Ontario water balance studies is the apparent bias in the procedure used to determine the Niagara River flows for publication. The published flows appear to be biased on the high side by approximately 2 percent. Pending additional measurements, it is recommended that the published Niagara River discharges for 1972 and 1973 be reduced by 1.9 percent for IFYGL water balance studies.

REFERENCES

Carter, R. W. 1970. Accuracy of current meter measurements. *Hydrometry, Proc. of the Koblenz Symp.* UNESCO, pp. 86-94.

Cox, P. L. 1974. Lake Ontario outflow measurements, July 1972. Presented at the 17th Conference on Great Lakes Research, August 1974.

Herschy, R. W. 1970. The magnitude of errors at flow measurement stations. *Hydrometry, Proc. of the Koblenz Symp.*, UNESCO, pp. 109-130.

Quinn, F. H. 1978. The derivation and calibration of stage-fall-discharge equations for the Great Lakes connecting channels (unpublished report).

______, and Wylie, E. B. 1972. Transient analysis of the Detroit River by the implicit method. *Water Resour. Res.* 5:1461-1469.

Water Survey of Canada. 1972. Lake Ontario inflow measurements, July 1972. A report for the International Field Year for the Great Lakes, Lake Ontario Terrestrial Water Balance Panel.