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DERIVATION AND CALIBRATION OF STAGE-FALL-DISCHARGE EQUATIONS FOR THE GREAT LAKES CONNECTING CHANNELS*

by

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1. INTRODUCTION

Stage-fall-discharge equations for the connecting channels of the Great Lakes, used for many years to compute monthly flow rates, are currently incorporated in hydrologic response models of the Great Lakes system. However, their derivation and calibration have not been properly documented in the literature. This report gives one derivation and explains how they are calibrated with discharge measurements.

2. DERIVATION

This derivation is based upon the application of Manning's equation to the connecting channels. Manning's equation is

$$Q = \frac{AR^{0.67} S^{0.5}}{n}, \qquad (1)$$

where Q is the flow in cubic meters per second,

A is the cross-sectional area in square meters,

R is the hydraulic radius defined as A/P in meters,

P is the wetted perimeter in meters,

S is the slope of the water surface, and

n is Manning's roughness coefficient in meters$^{-.33}$ per second.

Consider the definition sketch in figure 1. The figure represents a connecting channel with a reach of length L, an average top width T, and a depth d. The water surface elevations at the upper and lower ends of the reach are given by Z_1 and Z_2, respectively.

For wide, relatively shallow channels, the wetted perimeter P can be approximated by the top width T. For a typical channel with T = 305 m (1000 ft) and d = 9 m (30 ft), the error induced by this assumption would range from less than 1 percent, if the channel were parabolic, to about 6 percent, if the channel were rectangular. For many regular geometric channel configurations, the area A can be expressed as a linear function of the depth d and top width T, as shown by the following examples:

rectangle $A = Td$,

parabola $A = \frac{2}{3} Td$, and

triangle $A = \frac{Td}{2}$.

The hydraulic radius and area are now expressed as

$$A = cTd, \text{ and}$$

$$R = cd,$$

where c is a constant depending upon the channel configuration. Equation (1) can then be written as

$$Q = \frac{c^{1.67} \, Td^{1.67}}{nL^{0.5}} (Z_1 - Z_2)^{0.5}, \tag{2}$$

where $\frac{Z_1 - Z_2}{L}$ is the slope S of the water surface.

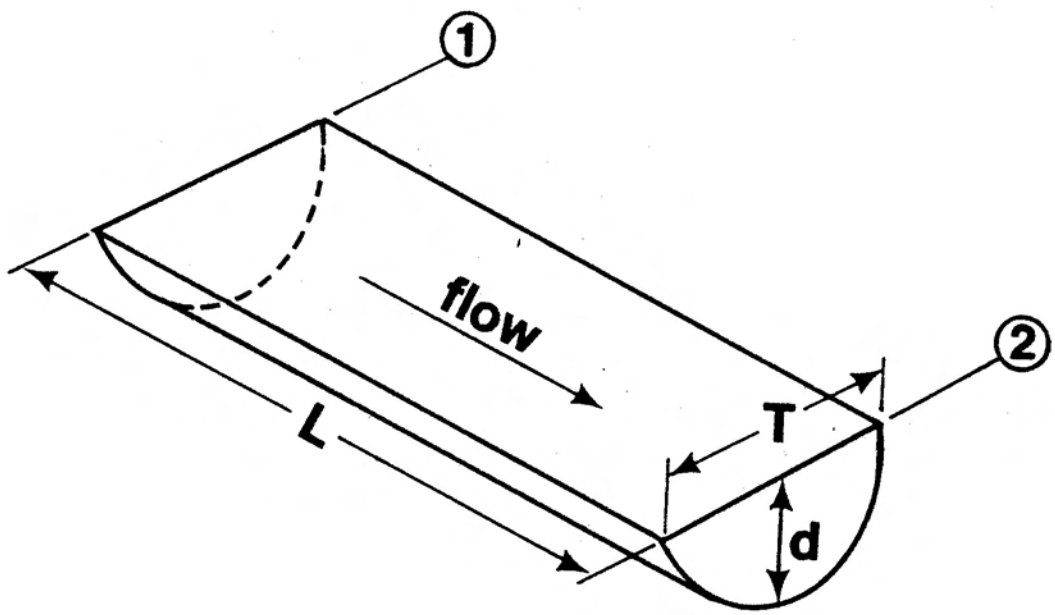

Figure 1. *Connecting channel definition sketch.*

The top width T can also be expressed in terms of the depth d by

$$T = bd^k. \tag{3}$$

The value of K• is dependent upon the channel configuration, being equal to 0 for a rectangular channel and 0.5 for a parabolic one. In unpublished work, the author found k to be approximately 0.3 for the St. Clair River. Substituting equation (3) into equation (2) with k of 0.3, we obtain

$$Q = \frac{c^{1.67}\ bd^{1.97}}{nL^{0.5}} (Z_1 - Z_2)^{0.5}. \tag{4}$$

Rounding the depth exponent to 2 and consolidating the coefficients into a single coefficient C, we obtain

$$Q = Cd^2(Z_1 - Z_2)^{0.5}, \tag{5}$$

where $C = \frac{c^{1.67}}{nL^{0.5}} b.$

Depth d can be expressed as

$$d = [f(Z_1,Z_2) - ym], \tag{6}$$

where ym is a coefficient known as the mean bottom elevation. Thus, equation (6) becomes

$$Q = C[f(Z_1,Z_2) - ym]^2(Z_1 - Z_2)^{0.5}. \tag{7}$$

Equation (7) is the general form of the stage-fall-discharge equation. Two of the most commonly used functions of depth are

$$f(Z_1,Z_2) = Z_1 \qquad \text{and} \qquad f(Z_1,Z_2) = 0.5Z_1 + 0.5Z_2.$$

In practice, either formulation is acceptable.

Coefficient C and constant ym are determined from discharge measurements. Equation (7) can be arranged as

$$f(Z_1,Z_2) = \left(\frac{1}{C}\right)^{0.5} \left(\frac{Q}{(Z_1 - Z_2)^{0.5}}\right)^{0.5} + ym. \qquad (8)$$

Equation (8) is linear in terms of $f(Z_1,Z_2)$ and $\left(\frac{Q}{(Z_1 - Z_2)^{0.5}}\right)^{0.5}$.

Values of ym and $\left(\frac{1}{C}\right)^{0.5}$ are determined by the least square solution. The required calibration data are measured discharges Q and corresponding water surface elevations Z_1 and Z_2.