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Spectral transmittance of lake ice from 400–850 nm

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Abstract

Spectral transmittance signatures for a variety of freshwater ice types were collected in the 400–850 nm range. Clear ice showed the highest transmittances, snow-covered ice the lowest and other ice types intermediate transmittances. All of the ice spectral transmittance curves showed the same general shape with no mutually-exclusive characteristics exhibited by any ice type. The magnitude of the transmittance values was the primary distinguishing factor. The range of transmittance values directly under the ice was remarkably narrow (excluding snow-covered ice) and markedly different from values lower in the water column.

Introduction

Much information is available on the transmittance of photosynthetically active radiation through sea ice and the associated effects on marine organisms under sea ice (SooHoo *et al.*, 1987; Palmisano *et al.*, 1987; Grenfell & Perovich, 1986; Gilbert & Buntzen, 1986; Perovich *et al.*, 1986; Grenfell & Maykut, 1977; Maykut & Grenfell, 1975; for example). Instrument systems have been developed specifically to measure the spectral transmittance of radiation through thick polar ice (Roulet *et al.*, 1974).

Such measurements are not available for freshwater ice and even a basic knowledge of the spectral transmittance of the various types of freshwater ice is sorely needed by biologists conducting freshwater under-ice ecology experiments. Measurements of photosynthetically active radiation (PAR) transmittance through freshwater ice have heretofore used sensors which integrate the radiation over the entire 400–700 nm range or portions of that range (Greenbank, 1945; Wright, 1964; Saijo & Sakamoto, 1964; Halsey, 1968; Goldman *et al.*, 1967; Anderson, 1970; Schindler & Nighswander, 1970; Schindler, 1974; Maguire, 1975a, b; Bolsenga, 1981; Stewart & Brockett, 1984; Roulet & Adams, 1984). However, it is known that freshwater zooplankton often adjust their position in the water column, not only according to the quantity but also the wavelengths of radiation (e.g., Beeton & Bowers, 1982). Measurements of PAR in small wavelength increments are thus as critical to understanding the behavior of freshwater biota as in the marine environment. In addition, quality, commercially manufactured underwater spectroradiometers have recently become available which, although larger than those described above, are well suited to freshwater ice transmittance measurements where the cutting of larger (60 cm square) access holes is feasible. In this study, the spectral transmittances of PAR through various types of common freshwater ice using commercially available spectroradiometers are described.

Instrumentation and methods

Measurements of irradiance were made using a Li-Cor, LI-1800, underwater spectroradiometer in a level position just under the ice/water interface (referred to throughout this paper as 'at the ice/water interface' or '0 m') and at various water depths below the ice. A holographic-grating monochromator; order-sorting filter wheel; and a stable, high quantum efficiency silicon detector provided rapid, automatic scanning over the 400–850 nm range in 2 nm increments.

The arm used to position the instrument under the ice consisted of sections of 2-inch aluminum pipe for vertical control and a hinged member to move the instrument to an undisturbed area 2 m away from an access hole in the ice. Floats attached to the instrument provided neutral buoyancy; the instrument was attached to the horizontal arm by a shackle which allowed free movement in a horizontal plane to achieve automatic leveling. Vertical movement to acquire readings at various depths was accomplished by adding additional sections of pipe to a depth of 5 m after which the unit was lowered by line to the bottom.

Measurement plans varied according to the situation, but typically a scan was taken in air; the instrument was lowered under the ice for a scan; and the instrument was subsequently lowered to depths of 1, 2, 3, 5, and 10 m under the ice surface and to the bottom. The sequence was then repeated in reverse with a scan in the air completing the operation. Total time for the scans at a single site required about 7–10 min. Measurements were normally completed under uniform sky conditions (sky completely clear or totally overcast). Ideally, two spectroradiometers should be used to record air and under-ice readings simultaneously. Data obtained where sky conditions were known to have affected the readings were not used.

Measurements were made on two inland lakes near Ann Arbor, Michigan, U.S.A. and on Lake Erie of the Laurentian Great Lakes (Table 1). The wide variety of ice types encountered include most of those that form on freshwater lakes.

Results

This and previous investigations (Maguire, 1975a, b; Bolsenga, 1981) show that snow has a greater effect on the diminution of incoming PAR than any other feature of the ice types studied. The ice surface at Frains Lake (Table 1) was completely covered with fluffy snow (10 cm thick at the measurement site) which showed little adhesion to the ice surface. The ice was 20 cm thick (7 cm of snow-ice over 13 cm of clear ice). Scans at the ice/water interface and at various depths below the ice surface are shown in Fig. 1. The maximal amount of radiation which penetrated the ice/snow combination was only 2.8% at the ice/water interface peaking at 530 nm. In the water column, the same pattern persisted with a noticeable diminution of radiation at about 675 nm. Maximal transmittance peaked within the narrow range of 588–592 nm for the five scans

Table 1. Measurement locations and descriptions.

Lake	Location	Surface area (sq km)	Max depth (m)	Depth at site (m)
Frains	(42°20′N, 83°37′W)	0.67	9	7.5
Bishop	(42°30′N, 83°51′W)	0.43	16.5	12.0
Erie	(41°39′N, 82°50′W)	3680.0*	14.0*	8.9–11.0

* Western Basin

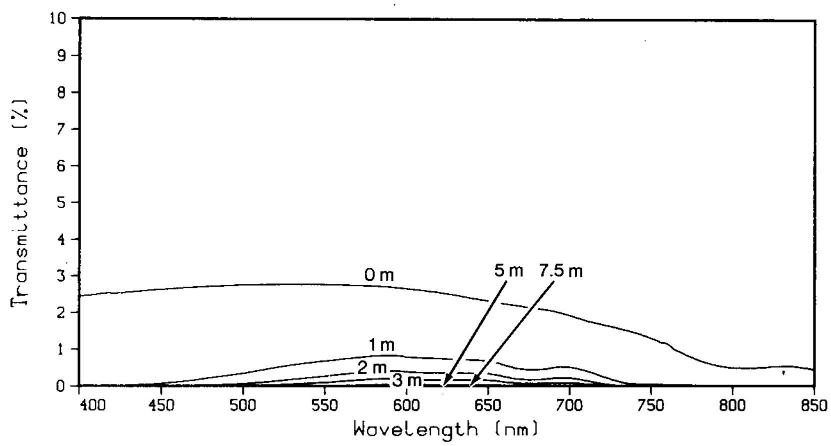

Fig. 1. Spectral transmittance measurements under snow-covered ice (Frains Lake). Note scale.

in the water column. Figure 2 shows four scans at Frains Lake; under the snow-covered ice and with snow removed from the ice over the sensor in areas approximately 60 cm, then to 1 m, then 2 m in diameter. The effect of the snow cover on the amount of transmitted radiation is apparent and indicates the possible deleterious effects on organisms which depend on light for survival. Other readings at three levels below the snow-cleared ice surface show the same pattern as those described under the snow-covered surface, but with lower transmittances.

Measurements were taken on Lake Erie through a 21 cm thick, partially snow-covered, white/clear ice combination. The amount of white ice varied widely within the ice, so that some areas were mostly clear ice while others were a full 13.2 cm of white ice. The surface of the ice varied from hard, wind-packed, smooth snow 7–8 cm thick, to snow-free ice exhibiting a whipped or wind-stippled surface. Results of scans at the ice/water interface under each of these surfaces (using the same access hole) are shown in Fig. 3. The effect of snow on the diminution of radiation is again apparent in these measurements. Transmittance of more than 85% through the 'clearer'/stippled ice is reduced to 2.5% or less by the thin snow cover. Transmittance is also significantly reduced under the stippled/white ice. Peak transmittances were 85.7% at 542 nm for

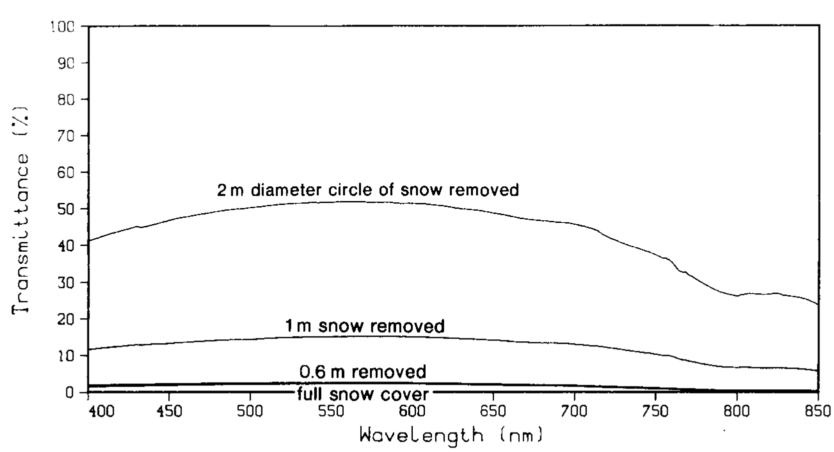

Fig. 2. Spectral transmittance measurements with snow progressively removed from ice surface over sensor (Frains Lake).

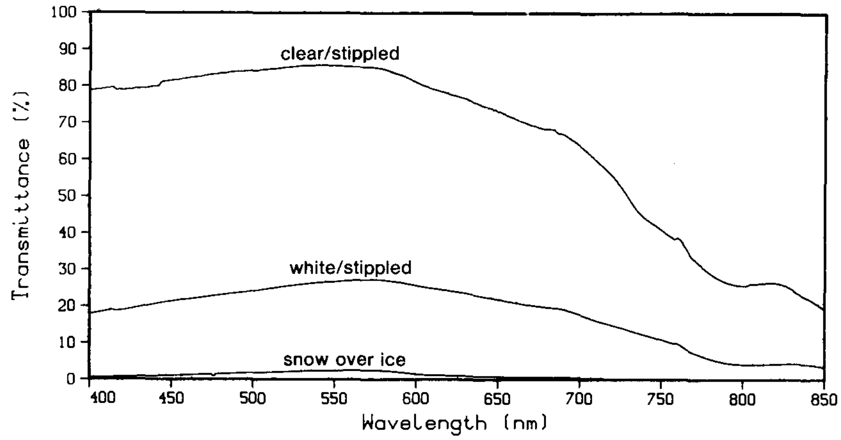

Fig. 3. Spectral transmittance measurements under snow-covered ice and nearby stippled ice (Lake Erie).

clear/stippled ice, 27.2% at 572 nm for white/stippled ice, and 2.5% at 564 nm for snow-covered ice.

Clear ice transmittances contrast sharply with snow-covered ice transmittances. Clear ice can transmit PAR in the 90% range, depending on ice thickness and structure. The clear ice studied here was located on Lake Erie and, though it exhibited an extensive bubble structure, the transmittances were the highest obtained for all of the ice types. Results of scans under 25.5 cm of clear ice with an overlying 8 cm layer of semi-cloudy ice and scans at various water depths below the ice are shown in Fig. 4. A maximal transmittance of 94.3% was recorded directly under the ice at 560 nm. Transmittances were high from 400–600 nm, then decreased steadily to 850 nm (except at about 825 nm in the surface and 1 m scans).

It is often impossible to distinguish between snow-ice (snow falling into open water which subsequently freezes; or snow loading an existing ice cover which cracks, admits water to the snowpack, then subsequently freezes) and refrozen slush (rain-soaked or wet snow which subsequently freezes). Without knowledge of their origins, these ice types are often referred to as white ice. White ice in the form of snow-ice is very common on smaller inland lakes. The shoreline of the lake is usually rimmed with trees that lower

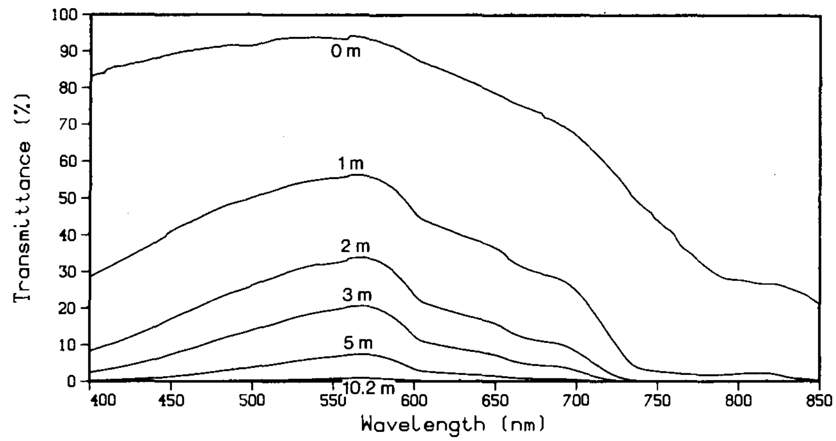

Fig. 4. Spectral transmittance measurements through clear ice (Lake Erie) and at various depths (in meters) below the ice surface.

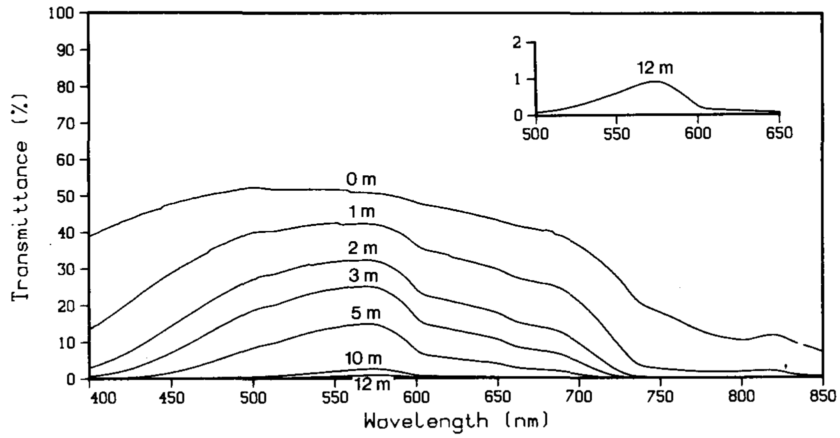

Fig. 5. Spectral transmittance measurements through snow-ice (Bishop Lake) on a clear day. Depths in meters below the ice surface. Inset shows bottom readings at a different scale.

the velocity of the wind and result in heavy accumulations of snow on the ice surface. White ice in the form of refrozen slush is common on the Great Lakes. At higher latitudes early in the season, heavy snowfalls and intervening above-freezing temperatures or rain often reduce snow cover on the ice to slush. At lower latitudes such events are common throughout the early and middle portions of the winter season. White ice can vary widely in structure (e.g. bubble concentration and size, layering, etc.) which makes definition of 'typical' white-ice-cover-transmittance difficult. Nevertheless, the measurements given here indicate the variations encountered.

Measurements were collected on Bishop Lake within a two-day period under the same ice cover, but with differing atmospheric conditions. The ice at the site was 28.6 cm thick (8.3 cm of snow-ice overlying 16.3 cm of clear ice with elongated air inclusions and 4.0 cm of relatively bubble-free clear ice at the bottom). The first series of scans was made under clear skies (Fig. 5). The shape of the scan-curves in water remains essentially the same with depth, but diminished in magnitude. As shown by the inset, a small amount of radiation reached to near the bottom. On the second day of measurements, the sky was overcast (only diffuse incident radiation). Measurements at the ice/-water interface on the clear and overcast days are compared in Fig. 6. Maximal differences between

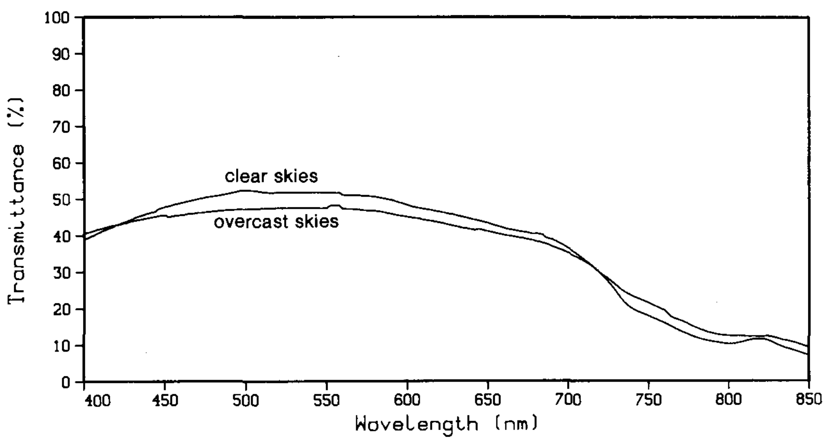

Fig. 6. Spectral transmittances under the same snow-ice surface (Bishop Lake) on a clear and a overcast day.

Table 2. Maximal transmittances and associated wavelengths through snow ice on a clear and an overcast day (Bishop Lake).

Clear skies			Cloudy skies		
T (%)	Wavelength (nm)	Depth (m)	T (%)	Wavelength (nm)	Depth (m)
52.4	500	0	47.6	534	0
42.7	552	1	42.1	562	1
32.5	558	2	30.8	566	2
25.2	568	3	22.8	566	3
15.0	570	5	13.6	568	5
2.6	572	10	3.2	572	10
0.9	574	12	1.3	572	12

the readings were only about 5%. Maximal transmittances and associated wavelengths for clear and overcast skies are given in Table 2. The only significant variation between the two days occurred in the upper 2 m, where transmittances were higher at lower wavelengths on the clear day. This is explained by the fact that diffuse radiation is relatively rich in visible light and snow-ice has a high reflectivity in the visible region (Bolsenga, 1983 and 1977). Transmittances under cloudy skies were slightly higher than those under clear skies at the 10 and 12 m depths, but these were not considered to be significant due to the small amounts of radiation penetrating to those levels.

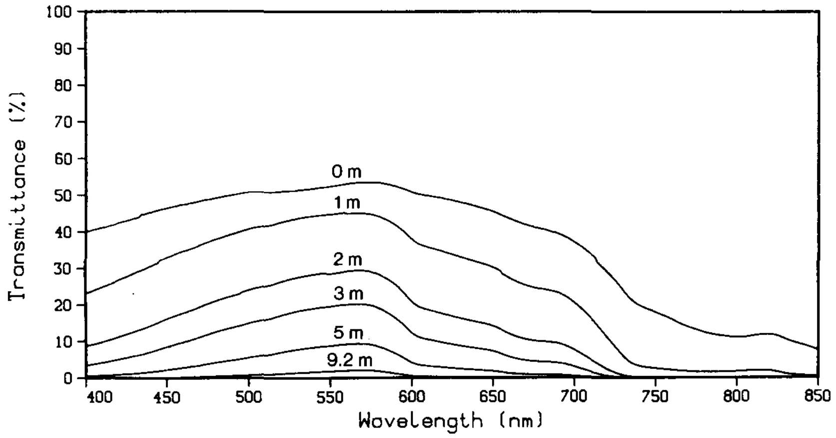

Fig. 7. Spectral transmittances at various depths (in meters) below a white ice layer (Lake Erie).

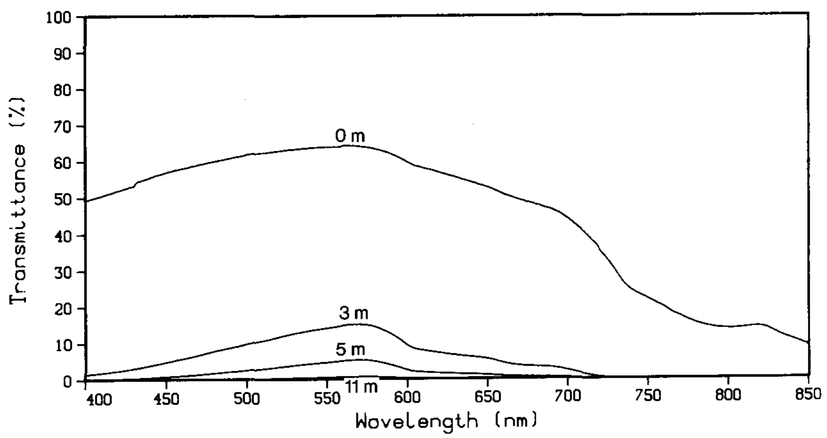

Fig. 8. Spectral transmittances at various depths (in meters) below a slush curd ice layer (Lake Erie).

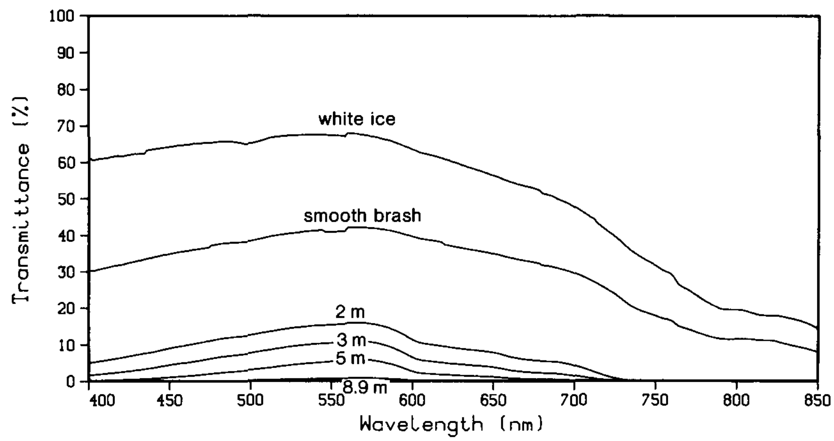

Fig. 9. Spectral transmittances through a smooth brash and adjacent white ice surface collected from the same access hole. Depths in meters below the ice surface (Lake Erie).

Measurements under an area of white ice on Lake Erie (24.5 cm thick and consisting of 18 cm of white ice atop 6.5 cm of clear ice) showed transmittances in the mid-range as compared to clear or snow-covered ice, with curves similar in form to those obtained at Bishop Lake (Fig. 7). Peak transmittance at the ice/water interface was 53.5% at 570 nm. The wavelength of maximal transmittance was 558 nm at 1 m, but remained at 566 nm throughout the remainder of the water column. Secchi depth was only 2.54 m; nevertheless, 2.2% of the incident PAR penetrated to the bottom.

Slush curd ice forms when a thick layer of slush freezes into a coagulated pattern of white ice in curd-shaped forms and intervening clear ice areas. Results of scans under 23.5 cm of slush curd ice with a 10 cm underlying clear ice layer are shown in Fig. 8. Measurements were taken under nearly clear skies with some thin, wispy clouds in areas other than on the solar beam. Maximal transmittance at the ice/water interface was 64.5% at 560 nm, after which the transmittance steadily dropped to about 10% at 850 nm. A rather steep decrease in transmittance was measured at 700–725 nm. In the water column, the wavelength of maximal transmittance remained in the narrow 568–570 nm range from 3 m below the ice surface to the bottom.

Brash ice is an accumulation of loose fragments of previously-formed ice randomly refrozen together. Scans were made from the same access

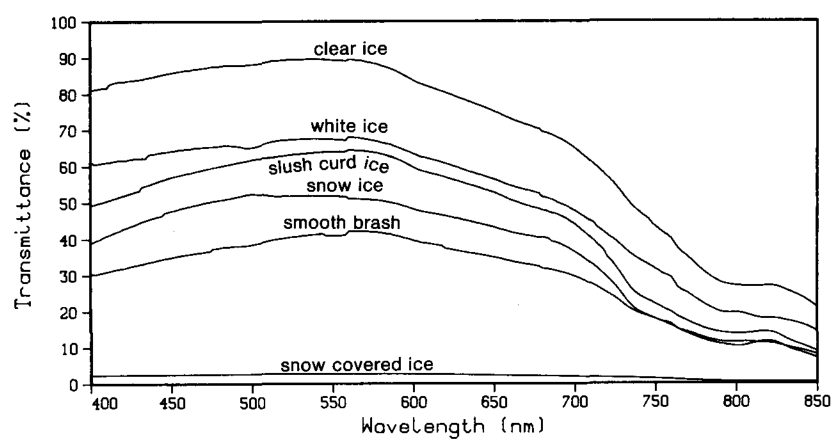

Fig. 10. A comparison of spectral transmittances at the ice/water interface of most of the ice covers measured in this study.

hole under Lake Erie ice which was 22.5 cm thick with a 13-cm-thick white ice upper layer overlying 9.5 cm of clear ice. The ice surface consisted of small chunks of ice frozen in a mildly chaotic pattern in one area near the access hole, and smooth ice in an adjacent area. Transmittances through each ice surface and at various water depths beneath the surfaces are shown in Fig. 9. The shape of the curves for both ice types at the ice/water interface was the same, with the greatest disparity in magnitude occurring at 400 nm. The disparity decreased steadily throughout the 400–850 nm range. Maximal transmittance for both ice types was at about 560 nm.

Studies conducted on the penetration of light through the water column in the Great Lakes (Jerome *et al.*, 1983 and Beeton, 1962), do not report measurements with an ice cover. Chandler (1942) collected data during the period of ice cover, but most of his measurements were made in open water areas in the ice cover with an instrument which measured only 'total visible light'. This study was not designed to collect irradiance profiles with depth; however, a limited number of measurements was made to determine if significant differences exist, due to the presence of ice, from those earlier studies.

Table 3 shows the spectral distribution of the radiation at various levels in the water column including directly under the ice. The spectrum is divided into three bands (e.g., Jerome *et al.*, 1983) defined as blue (400–500 nm), green (500–600 nm), and red (600–700 nm). Jerome *et al.* (1983) found that for Lake Erie the percentage of blue and red light decreased with decreasing subsurface irradiance, and the percentage of green light increased with decreased subsurface irradiance. Results from this study indicated that, excluding snow-covered ice, the range of the values obtained directly under the ice was remarkably narrow and markedly different from values lower in the water column. In the blue band the distribution of values at the ice/water interface ranged from 30.2–34.5%; green band, 35.4–38.7%; and red ban, 30.0–31.6%. This information should simplify predicting the effects of light transmittance through ice on biota directly under the ice, since the spectral effects appear to be dependent only on the amount of incident radiation.

The wavelength of maximal transmittance with depth varied slightly among the lakes studied. The peak of most transmittances directly under the ice ranged from 500–562 nm. Frains Lake data showed that the wavelength of maximal transmittance with depth varied from 584–588 nm at depths from 1 to 7.3 m. In Lake Erie the wavelength of maximal transmittance varied from 558–570 nm at depths ranging from 1 to 10.2 m. Most of the Lake Erie data showed maximal transmittances at 566 nm. Beeton (1962) found that radiation in the 590–610 nm range penetrated to the greatest depth on Lake Erie and in the 590–750 nm range on Frains Lake.

It is likely that water turbidity in the present study was significantly greater than during Beeton's measurements on Lake Erie. A storm occurred a few days before our measurements and the ice-water edge was only about 3000–4000 m from our measurement sites. The ice might not have been sufficiently extensive to act adequately as a cap during those storms and strong currents could have transported suspended sediments under the ice to the measurement area. Secchi depth on Lake Erie averaged 2.15 m for these measurements. At a site near our measurements, Holland (1989) measured Secchi depths of 3.5 m and above during the 1984 and 1985 ice covered periods. Chandler (1942) measured Secchi depths exceeding 4.5 m in western Lake Erie during the 1940 winter.

Jerome *et al.* (1983) found that the wavelength for maximal irradiance shifted from 550 to 570 nm for higher values of the irradiance attenuation coefficient and attributed this to increased concentrations of algal cells. Measurements on the clear day at Bishop Lake (Table 2) indicated maximal transmittance at 1 m was at 552 nm and increased to 574 nm at 12 m. On the overcast day, transmittance at 1 m was highest at 562 nm and increased to 572 nm through the water column. The only significant difference in measurements between the clear and the overcast day occurred in the upper 2 m of the water column.

Table 3. Spectral distribution of radiation for scans under the ice and at various water depths below the ice surface.

Location	Figure #	Depth m	Spectral distribution (%)		
			Blue (400–500 nm)	Green (500–600 nm)	Red (600–700 nm)
Frains	1	0.0	34.1	35.7	30.2
(Snw Cvd)		1.0	8.2	47.0	44.8
		2.0	2.3	47.0	50.7
		3.0	1.8	45.4	52.9
		5.0	1.0	45.0	54.0
		7.3 (BTM)	4.7	43.9	51.5
Frains	2	0.0	32.3	36.6	31.1
(Snw Remvd)		0.0	30.1	37.6	31.6
		0.0	31.3	35.4	33.2
		0.0	31.7	35.1	33.2
Bishop	5	0.0	33.2	36.2	30.1
(Snw Ice;		1.0	28.4	41.8	29.9
Clr Sky)		2.0	23.4	48.7	27.9
		3.0	19.8	55.0	25.2
		5.0	14.3	65.3	20.4
		10.0	4.5	82.3	13.2
		12.0 (BTM)	2.1	85.7	12.1
Bishop	6	0.0	33.8	35.5	30.7
(Snw Ice;		1.0	27.9	42.0	30.1
Dif Sky)		2.0	22.7	49.3	28.0
		3.0	19.5	55.5	25.3
		5.0	13.8	66.0	20.3
		10.0	4.1	83.0	12.9
		12.0 (BTM)	2.3	86.2	11.6
L. Erie	8	0.0	32.8	36.8	30.5
(Slush Curd)		3.0	22.0	55.3	22.8
		5.0	16.1	66.5	17.4
		11.0 (BTM)	8.5	82.2	9.3
L. Erie	4	0.0	34.4	35.6	30.0
(Clear Ice)		0.0	34.5	35.5	30.0
		0.0	34.1	35.7	30.1
		1.0	31.1	41.4	27.5
		2.0	27.2	48.5	24.4
		3.0	23.8	54.7	21.6
		5.0	18.1	65.6	16.4
		10.2 (BTM)	9.7	82.3	8.1
L. Erie	9	0.0	31.5	37.0	31.5
(W. Ice;		0.0	34.2	35.8	30.1
Smth Brsh)		2.0	28.9	46.8	24.4
		3.0	25.6	53.8	20.7
		5.0	18.6	65.2	16.2
		8.9 (BTM)	10.5	81.2	8.3

Table 3. (Continued).

Location	Figure #	Depth m	Spectral distribution (%)		
			Blue (400–500 nm)	Green (500–600 nm)	Red (600–700 nm)
L. Erie	3	0.0	27.2	54.0	18.7
(Clr & Wht		0.0	34.1	35.5	30.5
Ice-stippld; Snw cvd ice)		0.0	30.6	37.8	31.6
L. Erie	7	0.0	32.2	36.4	31.4
(Wht ice)		1.0	30.9	41.2	27.9
		2.0	28.4	47.4	24.2
		3.0	26.1	53.0	21.0
		5.0	18.8	67.6	13.6
		9.2 (BTM)	13.2	79.4	7.4

Discussion/Summary

The spectral transmittance signatures of a variety of ice types exhibited the same general shape from 400–850 nm. Clear ice showed the highest transmittance values across the spectrum and snow-covered ice the lowest. Other ice types, such as white and slush curd ice, showed transmittances somewhere between these two extremes. No exclusive distinguishing characteristic was found for any ice type or group of ice types with the exception of the magnitude of the transmittance values. Ice with a clear ice component (clear ice and slush curd ice) showed a plateau in transmittance decreases at about 825 nm, but this feature also appeared in the snow ice on Bishop Lake. Slush curd and the Bishop lake snow ice also exhibited a steep drop in transmittances between 700–725 nm. Figure 10 is a comparison of the under-ice-transmittances of most of the ice types measured.

The diminution of radiation by snow cover was apparent. The presence of even a thin snow cover reduced possible high transmittance of radiation through ice to very low levels. The effect of a heavy snowcover on under-ice life can obviously be drastic. Ice can have beneficial effects, however. With clear ice a large portion of the radiation can be transmitted to the water column, thus benefiting the biota. Ice also provides a cap on the water surface that reduces turbulence. Clear ice or even some forms of relatively opaque ice promote solar heating of the water column (Matthews & Heaney, 1987).

The spectral distribution of radiation at various levels in the water column was examined by dividing the spectrum into three bands. The range of values obtained directly under the ice was remarkably narrow and markedly different from values obtained lower in the water column. Changes of transmittances with depth in the water column generally agreed with previous studies.

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