nevertheless suggests the greater accuracy to be achieved by duplicate determinations.

While the values of the variability of temperature and salinity of the mixed layer are similar to those of the corresponding accuracies of measurement, the value for oxyty is almost twice as large (Table 3) as is indicated by the precision of the measurements. It is unlikely that biological activity in the layer could cause the oxyty to vary much. It is more likely that the larger variability has resulted from sampling errors. Even if the actual titration is carried out properly, errors could arise from a number of other sources, such as the condition of the interior of the hydrographic sampling bottles (which might be such as to promote oxidation), air introduced into the BOD bottles while the samples are drawn, and undue delay in adding reagents. There is no apparent reason why the variability of oxyty could not be reduced if proper procedures are followed during sampling and determination. The data so far obtained indicate that the layer is iso-oxygenic only to within ± 0.04 to ± 0.05 $ml \cdot liter^{-1}$ (Table 3). However, if we adopt the values resulting from duplicate determinations, it appears that the mixed layer can be considered iso-oxygenic to within ± 0.03 $ml \cdot liter^{-1}$ (Table 4)—a value close to the precision of the instrumentation.

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Applying phosphorus loading models to embayments[1]

Abstract—An explicit turbulent mass transport term is added to a phosphorus budget model to make it applicable to completely mixed water bodies with open boundaries; an application to Saginaw Bay demonstrates the importance of this modification. A method is devised to incorporate turbulent diffusion into loading plots by adjusting the loading term to include the flux of phosphorus from the main lake into the bay and including the effect of turbulent flow in the residence time.

Vollenweider, Dillon, and others have developed a general approach for predicting a lake's trophic state on the basis of simple expressions of its nutrient loading, morphology, and hydrology. The best known examples are phosphorus loading plots (Vollenweider 1968, 1975), which have found wide application in eutrophication analysis. Other expressions of the approach are nutrient budget models that parameterize some of the mechanisms underlying the process (Vollenweider 1969; Dillon and Rigler 1974; Chapra 1977). In most cases, the analyses have been of lakes where advection through river outlets is the predominant form of mass transport out of the system.

[1] GLERL contribution 133.

However, there is a large class of waters, namely embayments, where turbulent or diffusive transport is of considerable importance. For example, two of the most degraded water bodies in the Great Lakes are Saginaw Bay and Green Bay, which have large open boundaries with Lakes Huron and Michigan. I here develop a simple model for those bays that can be considered as completely mixed systems when a term for turbulent mass transport is added to a nutrient budget equation. The model is applied to Saginaw Bay where the importance of this term is demonstrated. A method of incorporating such effects into loading plots is also presented.

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A nutrient budget model for total phosphorus (Vollenweider 1969, as modified by Chapra 1975) can be written for the completely mixed lake in Fig. 1a as

$$V\frac{dp}{dt} = W + \sum_{i=1}^{n} Q_i p_i - Qp - vA_s p, \tag{1}$$

where V is volume of the lake, p is total phosphorus concentration of the lake, t is time, W is direct mass loading of total phosphorus to the lake, Q_i is flow of water into the lake from the ith tributary, p_i is the concentration of total phosphorus in the ith tributary, n is the total number of tributaries, Q is flow through the lake's outlet = the summation of all tributary flows if evaporation equals precipitation and the lake's volume is constant, v is the apparent settling velocity of total phosphorus, and A_s is the surface area of the lake's sediments. By grouping terms and assuming a steady state ($dp/dt = 0$), we can solve Eq. 1 for total phosphorus concentration,

$$p = \frac{W'}{Q + vA_s}, \tag{2}$$

where W' is the total loading to the lake = $W + \Sigma Q_i p_i$.

For systems having open boundaries with adjacent water bodies (Fig. 1b), however, an additional term must be added to Eq. 1 to account for the turbulent mass transport or mixing across the open boundary. Thomann (1963) has parameterized such transport as

$$\left(V\frac{dp}{dt}\right)_{\text{diffusion}} = E'(p_b - p), \tag{3}$$

where p_b is the concentration of phosphorus in the main lake; E' is the bulk diffusion coefficient, expressed as

$$E' = \frac{EA_c}{l} \tag{4}$$

(E is a turbulent diffusion coefficient, A_c is the cross-sectional area at the interface between the bay and the main lake, and l is the mixing length of the exchange process). Equation 3 is similar in concept to the tidal prism method (Phelps and Velz 1933; Ketchum 1950) designed to model flushing in salt water river estuaries where tidal oscillations are the predominant cause of turbulent exchange. In freshwater systems, seiches and transient currents would have an analogous effect of minimizing gradients and therefore causing mass transport between an embayment and a main lake. In fact, many investigators have used the method to estimate flushing times and the diffusion characteristics of freshwater bays (e.g. Beeton et al. 1967; Ahrnsbrak and Ragotzkie 1970). In the present context Eq. 1 and 3 can be combined to yield a nutrient budget equation for the idealized bay in Fig. 1b:

$$V\frac{dp}{dt} = W + \sum_{i=1}^{n} Q_i p_i - Qp - vA_s p + E'(p_b - p); \tag{5}$$

which, at steady state can be solved for

$$p = \frac{W''}{Q + vA_s + E'}, \tag{6}$$

where

$$W'' = W + \sum_{i=1}^{n} Q_i p_i + E' p_b. \tag{7}$$

Thus, the total loading to the bay system, W'', includes the loading due to turbulent

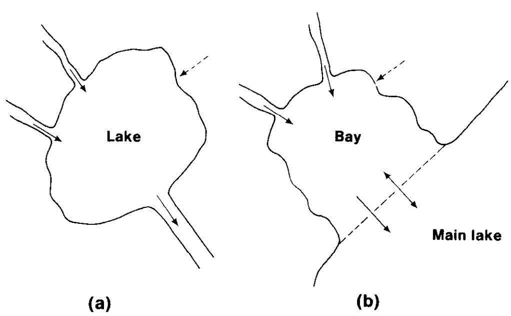

Fig. 1. Schematic overhead views of a lake (a) and an embayment (b) showing a waste input (-->), advective mass transport (→), and turbulent or diffusive mass transport (↔).

mixing from the main lake into the bay as well as tributary and direct inputs.

The bulk diffusion coefficient can be estimated theoretically or empirically. In the former case, this often involves the aggregation of current velocities calculated by numerical hydrodynamic models (Canale and Squire 1976). More commonly, where a sufficient gradient exists between the bay and the main lake, conservative substances such as chlorides can be used to estimate E' empirically (Ahrnsbrak and Ragotzkie 1970; Richardson 1976). This is done by writing a mass balance equation for the conservative substance as

$$V\frac{dc}{dt} = W_c + \sum_{i=1}^{n} Q_i c_i - Qc + E'(c_b - c), \quad (8)$$

where c is the concentration of conservative substance in the bay, W_c is the direct loading of conservative substance to the bay, c_i is the conservative substance concentration of the ith tributary, and c_b is the conservative substance concentration of the main lake. At steady state, Eq. 8 can be solved for

$$E' = \frac{W_c + \sum Q_i c_i - Qc}{c - c_b}. \quad (9)$$

An application of Eq. 6 and 9 to an actual system can be demonstrated for Saginaw Bay, a freshwater embayment in southwestern Lake Huron (Fig. 2). Chemical data were used to situate the interface between the bay and Lake Huron at the approximate location of an underwater ridge that forms a natural constriction or boundary between the two bodies of water. While gradients exist within the bay demarcated in this way, they are small (except in the immediate vicinity of the Saginaw River mouth) in comparison with the gradient across the interface. In addition the length of the gradient zone is small (10 km) relative to the length of the bay (55 km), which is a prerequisite of the assumption of complete mixing.

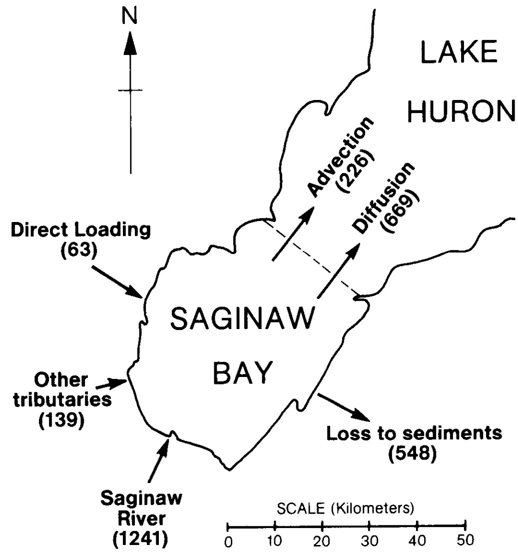

Fig. 2. Total phosphorus budget for Saginaw Bay (all values in $t \cdot yr^{-1}$).

Data for the system from 1974 through 1976 are summarized in Table 1. Substituting the appropriate values into Eq. 9 yields a bulk diffusion coefficient of 25.1 $km^3 \cdot yr^{-1}$. This can be expressed in the more familiar context of a turbulent diffusion coefficient by using Eq. 4. The resulting diffusion coefficient of about $5 \times 10^5\ cm^2 \cdot s^{-1}$ is generally of the order of horizontal diffusion coefficients for this scale of motion (Boyce 1974).

To calculate total phosphorus concentration, an estimate of the apparent settling velocity is required. Dillon (pers. comm.) has suggested a value of 12.4 $m \cdot yr^{-1}$, based on a least-squares fit of phosphorus budget data from a large

Table 1. Data for Saginaw Bay, 1974 through 1976 (from Smith et al. 1977; Upper Lakes Ref. Group 1977; Richardson and Bierman 1976; V. J. Bierman and D. Dolan pers. comm.; U.S. Geol. Surv. Rep. for State of Michigan).

Parameter	Symbol	Value
Saginaw Bay		
Volume	V	8.05 km^3
Surface area	A_s	1,376 km^2
Mean depth	z	5.85 m
Cross-sectional area	A_c	0.17 km^2
Mixing length	l	10 km
Outflow	Q	7.03 km$^3\cdot$yr^{-1}
Chloride concn	c	15.2 mg$\cdot$liter^{-1}
P concn	p	30.9 μg$\cdot$liter^{-1}
Direct loading chloride	W_c	negligible
Direct loading P	W	63 t$\cdot$yr^{-1}
Tributaries		
Saginaw River (i = 1)		
Flow	Q_1	5.73 km$^3\cdot$yr^{-1}
Chloride concn	c_1	56.4 mg$\cdot$liter^{-1}
P concn	p_1	216.6 μg$\cdot$liter^{-1}
All others (i = 2)		
Flow	Q_2	1.3 km$^3\cdot$yr^{-1}
Chloride concn	c_2	23.0 mg$\cdot$liter^{-1}
P concn	p_2	106.9 μg$\cdot$liter^{-1}
Lake Huron		
Chloride concn	c_b	5.4 mg$\cdot$liter^{-1}
P concn	p_b	5.5 μg$\cdot$liter^{-1}

number of lakes. While such an estimate should not be expected to apply to all lakes because of its statistical basis, it can provide a first estimate of sediment losses for Saginaw Bay. Substituting it into Eq. 6 results in a calculated total phosphorus concentration of 32.1 μg$\cdot$liter^{-1} that is close to the measured value of 30.9 in Table 1.

On the basis of the above calculations, a total phosphorus budget for Saginaw Bay can be constructed as in Fig. 2. Of the 1,443 t$\cdot$yr^{-1} coming into the bay from direct loadings and tributaries, 38% is incorporated into the sediments, 16% leaves by advection, and 46% by turbulent mixing. Thus, the inclusion of the last process accounts for nearly half the outflow of phosphorus.

Aside from nutrient budget models, the other major tool of the nutrient loading concept is the loading plot. As might be expected, the two are related (Chapra and Tarapchak 1976; Vollenweider 1976). For example, the steady state lake model (Eq. 2) can be rearranged to yield

$$W' = p(Q + vA_s). \qquad (10)$$

When we divide this equation by surface area and take its logarithm we get

$$\log L' = \log p + \log\left(\frac{z}{\tau_w} + v\right), \qquad (11)$$

since $Q/A_s = z/\tau_w$ (where z is mean depth of the lake), $\tau_w = V/Q$ (the lake's water residence time), and $L' = W'/A_s$ = the lake's areal phosphorus loading rate. Equation 11 is identical in form to one of the more recent Vollenweider (1975) loading plots which graphs the log of the areal loading vs. the log of z/τ_w. By choosing levels of phosphorus concentration that would be deemed "permissible" and "dangerous," we can construct a phosphorus loading plot from Eq. 11 that is quite similar to Vollenweider's (1975) plot.

In an analogous fashion, a loading plot for an embayment can be constructed on the basis of Eq. 6 to yield

$$\log L'' = \log p + \log\left(\frac{z}{\tau'_w} + v\right), \qquad (12)$$

where $L'' = W''/A_s$ and $\tau'_w = V/(Q + E')$. Thus, to incorporate the turbulent effect in the plots, a term representing the flux from the main lake into the bay must be included in the loading term (as in Eq. 7), and the residence time must be adjusted to reflect the turbulent flow.

For Saginaw Bay, the adjustment of the loading term represents a 10% increase from 1,443 t$\cdot$yr^{-1} (without diffusion) to 1,581 (with diffusion). The inclusion of turbulence reduces the residence time from about 1.1 years to 3.0 months.

The primary purpose of the foregoing presentation was to demonstrate the importance of including turbulent transport in embayment loading models. Since the Vollenweider approach has been developed primarily for lakes where advective transport through outlets is the major form of flushing, it is also intended as a warning against the casual application of phosphorus loading models to bays. This is particularly true for those models with a predominantly statistical basis. Lest the present note be similarly misapplied, it is important to stress some of its key assumptions and simplifications. These include—

Estimation of in-lake losses: In the preceding analysis I used Dillon's statistical determination of v to obtain a first estimate of sediment losses. Where a priori measurements of phosphorus concentration are available, an alternative approach would be to rearrange Eq. 6 to calculate the apparent settling velocity as

$$v = \frac{W'' - Qp - E'p}{A_s p}. \qquad (13)$$

For Saginaw Bay, this results in $v = 13.8$ $m \cdot yr^{-1}$. Such a calculation affords a direct estimate of in-lake losses for the system and could then be used in subsequent predictions.

Complete mixing: The foregoing derivation and application was limited to the case where the embayment is treated as a single completely mixed system. Such an idealization is only valid where the exchange process takes place in a zone that is short compared with the length of the embayment. As in Saginaw Bay, this is often predicated on the existence of a constriction that reduces mixing between the two bodies of water at a fairly well defined point along the axis of the bay. In many other cases, where the gradient is more continuous and the boundary with the main lake is not clearly delineated, a series of completely mixed segments would be required to adequately characterize the system. Mass balance equations could then be written for each segment and since the approach is linear and steady state, solutions are readily obtainable (Thomann 1972).

The analysis treated the main lake as an ocean, i.e. an infinite reservoir that is unaffected by changes in the bay. While this is a fairly realistic assumption when the main lake is as large as Lake Huron, smaller lakes may be highly dependent on their embayments. In such cases, an additional mass balance equation could be written for the main lake and solved simultaneously with the bay equation.

Temporal variability: As is typical of phosphorus loading models, my approach uses annual averages and does not resolve seasonal or diurnal variations. In essence, this represents an implicit assumption that the within-year variability is not a significant determinant of the annual prediction. Due to the unique role of turbulent mixing in their dynamics, embayments may differ from lakes in this respect. For example, Richardson (1976) suggested that differential heating of Saginaw Bay and Lake Huron effectively reduces the exchange between them in the spring. In smaller lakes, shifts in the prevailing wind patterns could have a similar effect. Such a phenomenon might have a marked effect at times of the year by trapping nutrients within the bay and enhancing phytoplankton productivity. Whether such an effect would bias the annual prediction is unclear and beyond the scope of this study. However, it suggests that in addition to annual average approaches, embayments and other physically dominated systems (e.g. fast flushing lakes) might require models that resolve features of their seasonal variability (e.g. peak phytoplankton values) for adequate characterization.

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A modification of the Hynes method for estimating secondary production with particular significance for multivoltine populations

Abstract—The accepted procedure for determining production of multivoltine invertebrates by use of the Hynes method is to multiply the Hynes value by the number of generations per year. For aquatic insects, if pupal, adult, or egg stages comprise a significant portion of total generation time, this procedure will underestimate production. For crustaceans, if reproduction occurs before attaining the final size class, the procedure, using generation time, will overestimate production. It is necessary to multiply the Hynes value by 365/*CPI*, where *CPI* is the cohort production interval (in days) from hatching to the attainment of the largest aquatic size class.

The Hynes method for measuring aquatic invertebrate production (Hynes 1961; Hynes and Coleman 1968; Hamilton 1969) has received considerable attention over the last few years. Unlike other methods which require recognition of individual cohorts (Waters 1977; Gillespie and Benke 1979), the Hynes method can be applied to less synchronous populations. Although the method was originally proposed for use on the entire benthic fauna taken together, it is now considered much more accurate to apply it to single species or similar species groups (Benke and Waide 1977; Waters 1977). As with any new method, the Hynes method has received criticism (Fager 1969; Zwick 1975), but it still appears valid as a reasonable approximation when specific growth data are unavailable (Hamilton 1969; Benke and Waide 1977).

The basis of the method is the calculation of an average cohort, or an average