GLERL

LAKES MICHIGAN - HURON OUTFLOWS

ST. CLAIR & DETROIT RIVERS

1900 - 1978

by the
Coordinating Committee on
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Hydrologic Data

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flow models and stage-fall discharge equations. The different methods used to determine flows for the period 1959-1978 are described in the following paragraphs.

METHOD USED BY GREAT LAKES ENVIRONMENTAL RESEARCH LABORATORY (U.S.)

41. Unsteady Flow Models. Hydraulic transient models were developed to simulate unsteady flow rates in the rivers. These models can be operated at hourly, daily, or monthly time intervals. The models are based upon complete partial differential equations of continuity and motion, expressed in terms of flow "Q" and stage "Z" above a fixed datum as follows:

$$\frac{dZ}{dt} + \frac{1}{T}\frac{dQ}{dX} = 0 \qquad (1)$$

$$\frac{1}{A}\frac{dQ}{dt} - \frac{2QT}{A^2}\frac{dZ}{dt} + \left(g - \frac{Q^2T}{A^3}\right)\frac{dZ}{dX} + \frac{gn^2Q/Q/}{2.208\ A^2R^{4/3}} = 0 \qquad (2)$$

where

X = discharge in the positive flow direction

t = time

A = channel cross-sectional area

T = top width of the channel at the water surface

g = acceleration due to gravity

R = hydraulic radius

n = Manning's roughness coefficient

d = partial derivative function

/ / = absolute value.

42. Equations (1) and (2) were placed in finite difference form at point M in an X-t grid (see Figure 10) to yield respectively,

$$\frac{Zu' + Zd' - Zu - Zd}{2\ \Delta t} - \frac{\theta\ (Qd' - Qu') + (1-\theta)\ (Qd - Qu)}{T\ \Delta X} = 0 \qquad (3)$$

$$\frac{Qu' + Qd' - Qu - Qd}{2\ \bar{A}\ \Delta t} - \frac{\bar{Q}T\ (Zu' + Zd' - Zu - Zd)}{\bar{A}^2\ \Delta t} +$$

$$\left(g - \frac{\bar{Q}^2 T}{\bar{A}^3}\right) \cdot \frac{\theta\ [(Zd' - Zu') + (1-\theta)(Zd-Zu)]}{\Delta X} +$$

$$\frac{gn^2\ \bar{Q}|\bar{Q}|}{2.208\ \bar{A}^2 R^{4/3}} = 0 \qquad (4)$$

where prime indicates LOCATION and overbars indicate MEAN, such that

$$\theta = \frac{\Delta t'}{\Delta t}$$

$$\bar{Q} = 0.5\ [\theta\ (Qu' + Qd') + (1-\theta)\ (Qu + Qd)]$$

$$\bar{A} = 0.5\ [\theta\ (Au' + Ad') + (1-\theta)\ (Au + Ad)].$$

43. Solution of equations (3) and (4) by the implicit method forms the basis of the transient models. A stable solution for these equations is provided by the weighting coefficient θ, which was selected empirically to be 0.75. Application of the equations at sections within pre-determined reaches produces a set of nonlinear equations that are solved simultaneously with linear approximations by the Newton-Raphson numerical iteration procedure. Descriptions of the employed St. Clair and Detroit River models, including calibration, sensitivity analysis, program listings, and output samples are given by Quinn & Wylie(12) and Quinn and Hagman(11). These initial models for both rivers have been revised to

include various improvements. The modified St. Clair River models are described by Derecki and Kelley[1] and for the Detroit River by Quinn[7][8].

44. <u>Transfer Factors</u>. Monthly hydrologic transfer factors pertaining to Lake St. Clair, for the period 1959-1976, were developed to enable comparison between the St. Clair and Detroit Rivers monthly flows. This transfer factor represents the hydrologic water balance for Lake St. Clair. Ignoring the ground water flux at the lake, which is assumed to be negligible, the transfer factor "T" is defined by the equation

$$T = P + R - E - S$$

where

P = over-lake precipitation
R = drainage basin runoff
E = lake surface evaporation
S = change in lake storage.

45. The above input parameters were determined independently from available data. The procedure is documented by Quinn[9]. Applying the transfer factor to the Lake St. Clair hydrologic balance yields the flow comparison equation

$$Q_{SC} + T = Q_D$$

where

Q_{SC} = inflow into lake from the St. Clair River
Q_D = outflow from lake into Detroit River.

46. St. Clair River-Open Water Flows. Four operational St. Clair River models, based on the one-dimensional equations for continuity and motion described earlier, were developed. These models span the upper portion of the river from its outflow at Port Huron to the City of St. Clair. Four U.S. water level gauges located along this reach supplied data for the models, three gauges per model; the midstream gauge data were incorporated for checking flow values by comparing computed and measured water levels. Each model provided three sets of flows corresponding to the river profile as indicated by the water level records of the employed gauges. The following models result from three-gauge combinations of the four selected gauges:

Fort Gratiot - Mouth of Black River - Dry Dock
Fort Gratiot - Mouth of Black River - St. Clair
Fort Gratiot - Dry Dock - St. Clair
Mouth of Black River - Dry Dock - St. Clair.

47. St. Clair River-Winter Flows. The same four models were used to compute winter flows. However, during the winter there is generally less agreement between the different St. Clair River models and frequent discrepancies occur between the St. Clair River and Detroit River flows. The discrepancy between the models is due to ice retardation of flows, which occurs quite often, especially in the lower St. Clair River. Resolution of the ice retardation problem requires winter flow measurements for model calibration; i.e., during periods when ice retardation causes significant changes in the normal open-water river profile.

48. Winter flows for the St. Clair River are computed by basically the same procedure used during open-water periods. Although some consideration is given to the transferred Detroit River flows, the St. Clair River models produce flows that are normally assumed to be more representative of actual conditions. This assumption is based upon the minimum flow criteria established by the Regulation Subcommittee, International Great Lakes Levels Board(13).

49. Detroit River-Open Water Flows. Two different transient models were developed. One is the upper river model, which is similar to the St. Clair River models. The other is the total river model, which branches into two channels in the lower portion of the river to give separate flows around Grosse Ile. Operation of the models for both rivers is similar, except that the total Detroit River model provides four additional flow values, corresponding to the upstream and downstream sections of the branching channels. The three-gauge relationships for the respective models are as follows:

Windmill Point - Fort Wayne - Wyandotte
Windmill Point - Wyandotte - Fermi.

50. Detroit River-Winter Flows. Both open-water models were utilized to compute winter flows, but the upper river model is considered more reliable, since it spans what is normally an ice-free reach. However, during periods when discrepancies occur between computed flows for both rivers, the recommended flows were based primarily on the transferred St. Clair River flows under the minimum flow criteria mentioned previously (St. Clair River-Winter Flows, paragraph 48).

METHOD USED BY THE U.S. ARMY CORPS OF ENGINEERS

51. Two-gauge stage-fall-discharge relationships were developed for various gauge combinations in both the St. Clair and Detroit Rivers. The equations were calibrated using recorded water level data and measured discharge data. Particular emphasis was placed upon inclusion of relationships in the ice-free reaches of both rivers to obtain reasonable estimates of winter flows. The basic equation is of the form:

$$Q = K\,(H-Ym)^a (F)^b$$

where

K = a constant

Ym = mean bottom elevation

F = fall in elevation between the two gauges

a = mean depth exponent

b = fall exponent

H = upper gauge elevation.

52. The rationale for this form of equation is given by Quinn(10). Studies and analyses through the years, by the governments of the United States and Canada, yielded the mutually agreed upon values of a = 2.0 and b = 0.5 (St. Clair River) and b = 0.4 (Detroit River). Solutions for the constant "K" and mean bottom elevation "Ym" were obtained through analyses of the average water levels and measured flows for the period of record. The water levels included data from all the permanent gauge sites between Lakes Huron and Erie, which were operating during the periods of flow measurements.

53. Later, the equation was expanded to the form:

$$Q = K\,[(P\,(Hu) + (1 - P)(Hd) - Ym)]^a\,(Hu - Hd)^b$$

where

P = a weighting factor ie .1, .2, .3 to .9.

Hu = water level at the upstream gauge

Hd = water level at the downstream gauge

Ym = hydraulic elevation at the river bottom.