

TESTING PARAMETRIC CORRELATIONS FOR WIND WAVES IN THE GREAT LAKES

Paul C. Liu

Great Lakes Environmental Research Laboratory/NOAA
2300 Washtenaw Avenue
Ann Arbor, Michigan 48104

ABSTRACT. A large number of wind and wave data recorded from eight NOMAD buoys in the Great Lakes during 1981 were used to examine correlations of wind wave parameters—nondimensional energy ϵ , nondimensional peak energy frequency ν , non-dimensional fetch ξ , and significant wave slope ss , among others—in detail for their universality. The results show no precise universal relations among the parameters. The correlations show that all the points are clustered around a distinct region rather than a linear regression line. The variations within the region can be up to one order of magnitude. It has been found that there are no clear seasonal or atmospheric stability effects on the correlations. Distinctive correlations are exhibited by lower and higher wind speeds. High values of ϵ are actually caused by low winds speeds rather than high waves. Individual episodes are examined. The correlations vary significantly among episodes. In practical applications, the only correlation that shows consistency and a smaller clustered region is that of ϵ versus ν , which has been used successfully in numerical parametric wave model development.

ADDITIONAL INDEX WORDS: Mathematical models, statistical methods.

INTRODUCTION

Wind wave studies, especially those linking theoretical predictions to measurements, frequently have to rely on empirical correlations of nondimensionalized parameters. Sverdrup and Munk (1947), for instance, developed a theory for wave growth that led to the functional relationships between non-dimensional fetch and wave age or nondimensional wave height that were supported by the available wave data at the time. Later Kitaigorodskii (1961) pointed out that all wave variables, when nondimensionalized in terms of acceleration of gravity g and wind speed U , should be functions of the nondimensional fetch gF/U^2 where F is fetch distance. Field and laboratory measurements appeared to support this contention. More recently, in the JONSWAP study, Hasselmann *et al.* (1973, 1976) inferred that, to incorporate the nonlinear energy transfer theory in a numerical wave prediction model, the full energy transport equation needs to be parameterized and the problem is reduced to the prediction of these parameters.

For a given wave field with spectral density $S(f)$, peak energy frequency f_m , and total energy $E = \int S(f)df$, along with wind speed at 10 m anemometer height and fetch F , the following parameters, among others, have been used in the literature:

ϵ	$= gE/U_{10}^4$	Nondimensional energy
ν	$= f_m U/g$	Nondimensional peak-energy frequency
ξ	$= gF/U_{10}^2$	Nondimensional fetch
SS	$= 2\pi f_m^2 E^{1/2}/g$	Significant wave slope (Huang <i>et al.</i> 1981)
α		Equilibrium range constant (Phillips 1958, 1977)
γ		JONSWAP peak enhancement factor (Hasselmann <i>et al.</i> 1973)
λ	$= \epsilon\nu^4\alpha$	JONSWAP shape factor (Hasselmann <i>et al.</i> 1976)
$\epsilon\nu/\xi$		Wallops fetch factor (Huang <i>et al.</i> 1981)

The basis of using nondimensional parameters is the expectation that by correlating these parameters universal relations can be found and thereby used in conjunction with theoretical models. Numerous studies have been made and a number

of universal correlations among the above parameters have been proposed and widely used. A relevant question seems to have escaped most of the attention in the literature: How universal are these universal correlations? In this paper, we attempt to explore this question by examining wave and wind data recorded from eight buoys in the Great Lakes during 1981. These Great Lakes data, averaging over 4,000 data points from each buoy, are ideal for studying the wind wave parametric correlations because the lakes are clearly fetch limited and generally free from swell complications.

THE DATA

Since 1979 the NOAA Data Buoy Center (NDBC) has deployed NOMAD buoys in the Great Lakes for long-term surface wave and meteorological measurements. Currently there are eight NOMAD buoys in Lakes Superior, Michigan, Huron, and Erie (Fig. 1), transmitting nearly real-time wind and wave information during the ice-free period of the year. The 6-m-long boat-shaped NOMAD buoys are moored in water depths ranging from 15 m in Lake Erie to 250 m in Lake Superior. The buoys are equipped to measure air and surface water temperatures, wind speed and direction at 5 m above the water surface, and wave spectra. The waves are measured with an accelerometer using an on-board Wave Data Analyzer System (Steele and Johnson 1977) that transmits acceleration spectral data via satellite to a shore collecting station where wave frequency spectra with 24 degrees of freedom are calculated from 20 min of measurements hourly. The wind speed and direction, as well as air and surface water temperatures, are measured with 1 s resolution and averaged hourly over 8.5 min data. In this study the measured wind speeds at a 5 m level are converted to

U_{10} based on the formula for overwater roughness length presented by Charnock (1955) and formulation for stability length presented by Businger *et al.* (1971), assuming the neutral drag coefficient to be 1.6×10^{-3} . The data used in this study were those recorded during 1981 and archived at the NOAA National Climatic Center in Asheville, N.C.

THE CORRELATIONS

Most of the studies in the literature are based on rather limited data points. JONSWAP, with over 300 data points, perhaps constitutes the most comprehensive study to date. The 1981 NDBC data provided us with over 33,000 data points from eight buoys; these are more than sufficient for our study. With a large number of continuous measurements, it is inevitable that there will be a large number of repetitious data points. It is certainly impractical to present all 33,000 data points; therefore we have chosen the eastern Lake Superior buoy 45004 for our presentation. This buoy, moored at a depth of 250 m, has recorded up to 7 m significant wave heights and 18 m s^{-1} wind speeds during 1981 among 3,665 data points. We have found, however, that the results from other buoys are essentially similar to those from buoy 45004, including the relatively shallow Lake Erie buoy 45004. Therefore, while the following correlations are based on buoy 45005 data, they are representative of all the Great Lakes.

Correlations with Fetch Parameters

One of the main goals of making empirical parametric correlations is to link internal and external parameters and thereby deduce the influence and range of external conditions on wave growth processes. The nondimensional fetch ξ is a successful and widely used external parameter. The fetch distance is in general taken as upwind distance from which the wind is blowing. In this study we define F as the average of all the fetches within $\pm 15^\circ$ of the wind direction to take care of the lake boundary effect. During JONSWAP, Hasselmann *et al.* (1973) presented the following power-law relations based on their own and a variety of other data sources, both field and laboratory measurements:

$$\nu = 3.5\xi^{0.33} \quad (1)$$

and

$$\epsilon = 1.6 \times 10^{-7}\xi \quad (2)$$

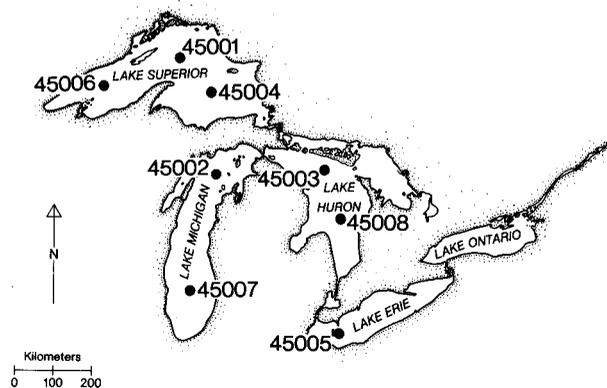


FIG. 1. Great Lakes map showing the locations of the eight NDBC NOMAD buoys.

to show the fetch dependence of the nondimensional peak frequency ν and nondimensional energy ϵ .

Contending that the balance of dynamical processes is different between field and laboratory, Phillips (1977) proposed an exponent of $-1/4$ in Eq. (1) based on field data only. Huang *et al.* (1981), however, chose to unify field and laboratory data by proposing the correlation of significant wave slope SS with a combined fetch parameter $\epsilon\nu/\xi$ given by

$$SS = (80\pi C_D^{-1} \epsilon\nu\xi^{-1/9})^{4/9}, \quad (3)$$

where C_D is the drag coefficient. Here we used the JONSWAP value of 1.0×10^{-3} for C_D .

Correlation of the parameters given in Eqs. (1)–(3) with the 1981 NDBC data is shown in Figure 2. If the relations are indeed universal, the 3,665 data points should be clustered around these straight lines. We found, however, that the points are clustered around galaxy-like regions instead. The variations within each region can be up to one order of magnitude. Since the regions generally follow the orientation of the respective straight lines given by Eqs. (1)–(3) and the lines go through the main cluster of points, the relation of Eqs.

(1)–(3), while not exactly universal, does represent a crude approximation of the correlations.

Since the 3,665 data points are applied nondiscriminately, we attempted next to sort the different external conditions to see if we can reduce the large scatter. We found there are no seasonal effects. The scatters are essentially similar for all seasons. Although Liu and Ross (1980) and others found that atmospheric stability affects wave growth, we found no discernible effects due to air-water temperature differences over this data set. We did find, however, that wind speed has a distinctive influence on the correlations. In Figure 3, we used only those data with wind speed greater than 10 m s^{-1} and those less than or equal to 2 m s^{-1} . These two groups of data occupied clearly separated regions in the correlations. The higher ν , lower ϵ , and lower ξ are for data with high wind speeds. The lower ν , higher ϵ , and higher ξ , on the other hand, are for data with low wind speeds. The same effect is not distinguished in the $SS-\epsilon\nu/\xi$ correlation. The established relations appear to fit the high wind speed data better than the lower wind speed data.

The separation of the data shown in Figure 3 can be envisaged as a result of the general definition of

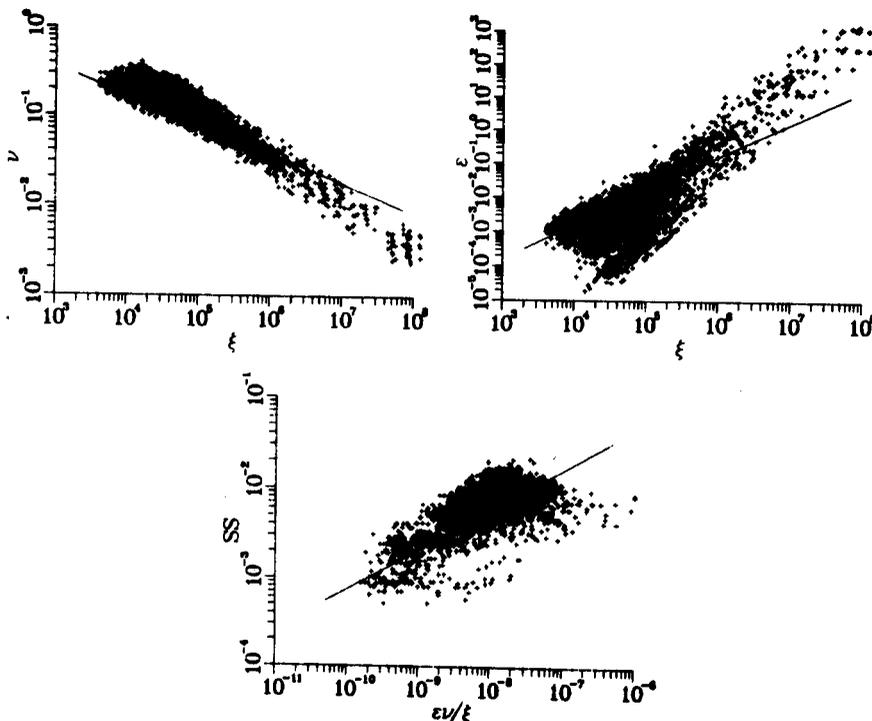


FIG. 2. Correlations of ν versus ξ , ϵ versus ξ , and SS versus $\epsilon\nu/\xi$. The lines in each graph correspond to Eqs. (1), (2) and (3), respectively. All data from buoy 45004 are used.

the nondimensional parameters. Since fetches do not vary significantly, the values of ξ for wind speeds 2 m s^{-1} and 10 m s^{-1} vary by an order of magnitude. Therefore, unless there are significant variations in fetch distance, wind speed is really the most important parameter of the correlations.

Continuing our efforts of reducing the scattering, we found that over 75% of the data points are waves with significant wave height under 1 m. In Figure 4, we used 880 data points with significant wave height greater than 1 m to yield fairly agreeable correlations. In subsequent correlations, we continued the use of these 880 higher wave data points with the understanding that lower wave data points would only contribute to increasing the already excessive scatter.

Correlations with Phillips' α

Phillips (1958, 1977) deduced from similarity considerations that the deep water wave spectra in the equilibrium range should be of the form

$$S(f) = \alpha g^2 f^{-5} \quad (f \gg f_m), \quad (4)$$

where α is a universal constant. In JONSWAP, Hasselmann *et al.* (1973) computed α by

$$\alpha = (0.65 f_m)^{-1} \int_{1.35 f_m}^{2 f_m} (2\pi)^4 f^5 g^{-2} \exp\left[\frac{5}{4}\left(\frac{f}{f_m}\right)^4\right] S(f) df \quad (5)$$

and again found that α is a function of nondimensional fetch ξ

$$\alpha = 0.076 \xi^{-0.22}. \quad (6)$$

Hasselmann *et al.* (1976) deduced that α is also a function of the non-dimensional peak frequency ν

$$\alpha = 0.033 \nu^{2/3}. \quad (7)$$

Both α and ν have been used as important parameters in developing numerical wave prediction models (Hasselmann 1977).

Other forms of the power law α versus ν relation have also been proposed. Toba (1978) used

$$\alpha = 0.44 \nu. \quad (8)$$

Mitsuyasu *et al.* (1980), on the other hand, developed

$$\alpha = 0.0326 \nu^{6/7}. \quad (9)$$

Figure 5 illustrates the correlations corresponding to Eqs. (6)–(9). Because the figures contain only data points with significant wave height

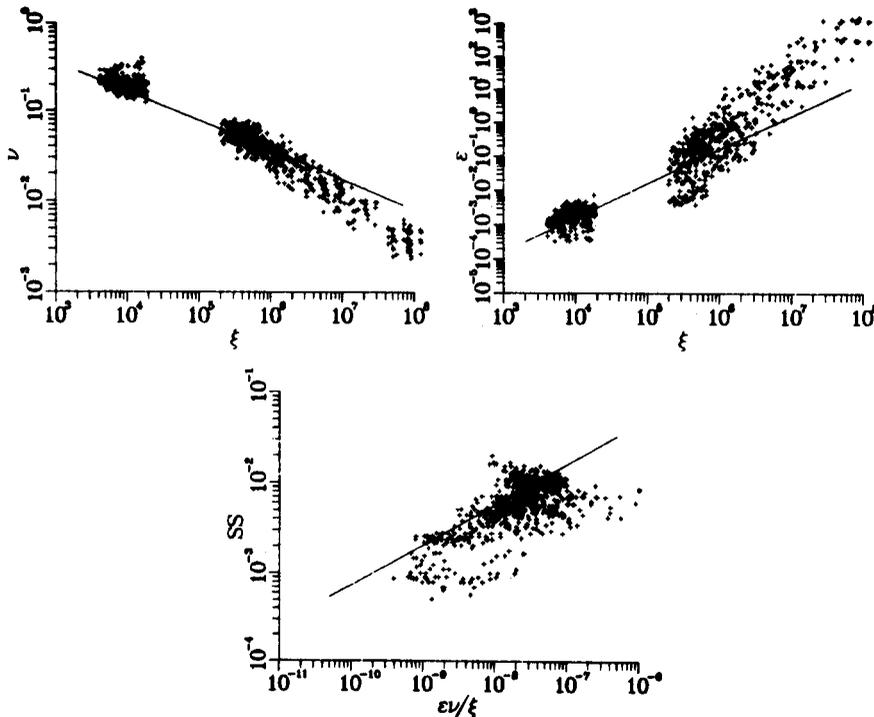


FIG. 3. Same as Figure 2, except that the data with wind speed greater than 10 m s^{-1} and less than 2 m s^{-1} are used.

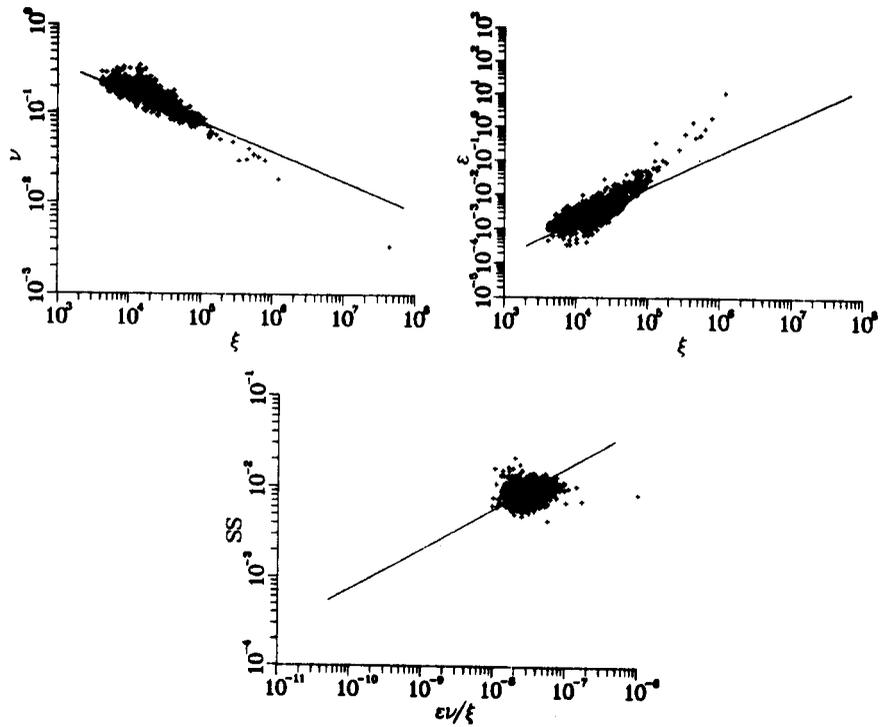


FIG. 4. Same as Figure 2, except that the data with significant wave height greater than 1 m are used.

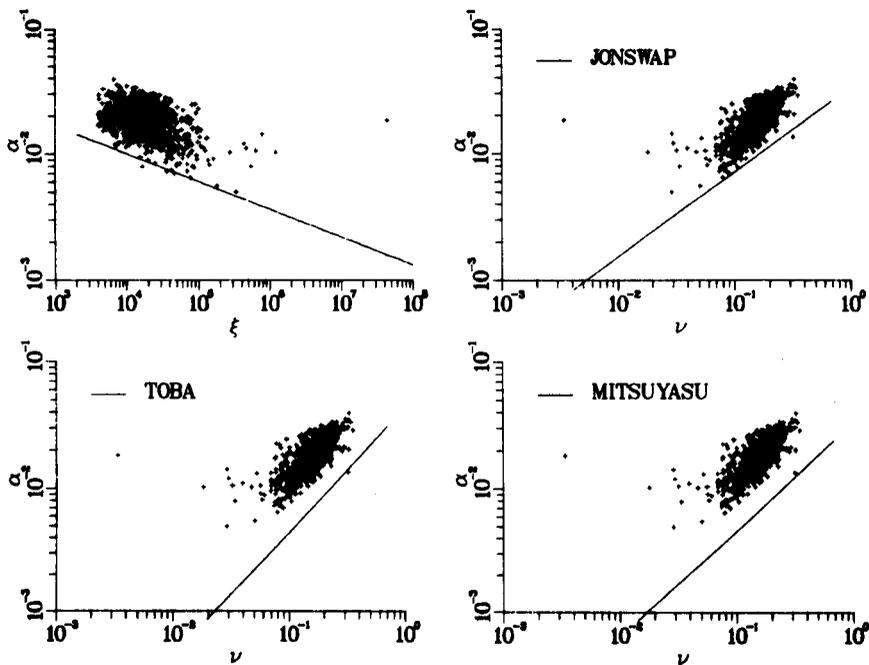


FIG. 5. Correlations of α versus ξ with Eq. (6) plotted and α versus ν with Eqs. (7), (8), and (9) plotted for JONSWAP, Toba, and Mitsuyasu, respectively.

greater than 1 m, they may indicate that the proposed relations are underestimating the data. However, when all the data points are plotted, they are too scattered to confirm the universality of any of the relationships and too scattered to warrant an examination of the differences among the three equations, (7)–(9).

Correlations with Shape Factors

The shape of the wave spectrum is an important parameter for wave predictions. Hasselmann *et al.* (1983) introduced the peak-enhancement factor γ , which is the ratio of the peak value of the spectrum to the peak value of the corresponding fully developed spectrum, which can be computed by

$$\gamma = S(f_m)(2\pi)^4 f_m^5 \exp(5/4)(\alpha g^2)^{-1}. \quad (10)$$

The contention is that γ is greater than 1 for growing waves and approaches 1 for fully developed waves. Hasselmann *et al.* (1976) defined an alternative shape parameter

$$\lambda = \epsilon \nu^4 / \alpha, \quad (11)$$

which is less affected by individual variations of the peak shape more dependent on average spectral properties. They used $\lambda = 1.6 \times 10^{-4}$ which is independent of other parameters.

Mitsuyasu *et al.* (1980) examined these parameters and deduced that

$$\gamma = 4.42 \nu^{3/7} \quad (12)$$

and

$$\lambda = (2\pi)^{-4} \gamma^{1/3} / 5. \quad (13)$$

Figure 6 represents a comparison of the data with the relationships (11)–(13). We found that λ can vary by an order of magnitude or more; γ versus ν correlates poorly; λ versus γ seems to have some correlation, but differs from Eq. (13). Furthermore the JONSWAP contention that fully-developed waves are those with $\gamma = 1$ and $\nu < 0.14$ is unrealized from these data. With many data having $\gamma < 1$ and $\nu < 0.14$, the data seem to be indifferent to the significance of these demarcations. These results cast doubt on the existence of

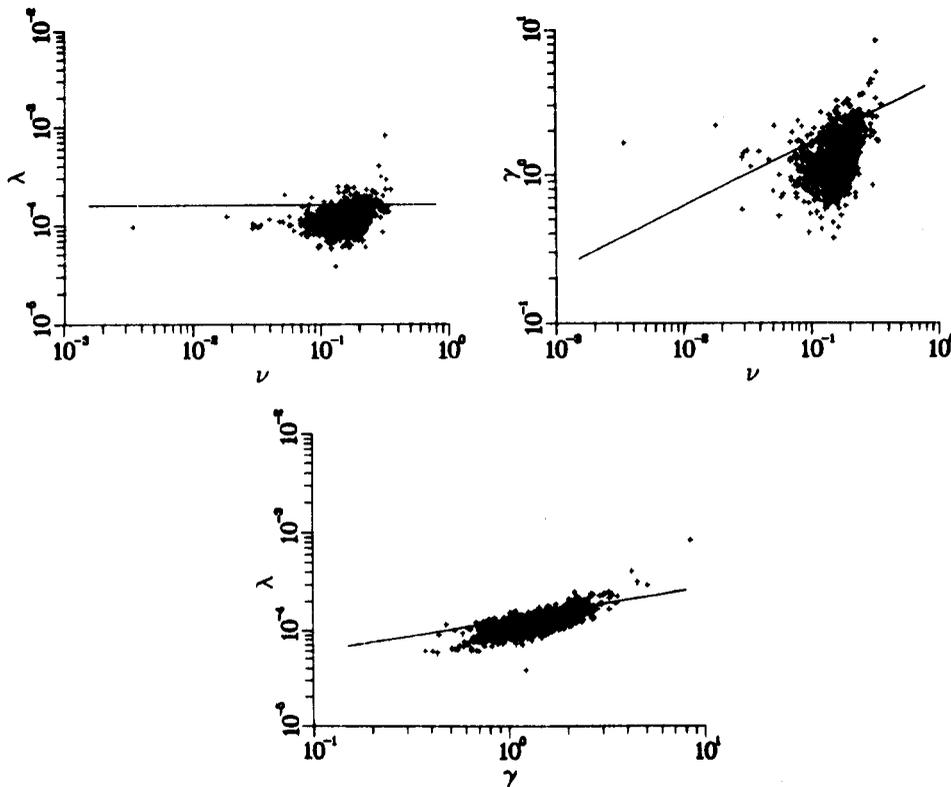


FIG. 6. Correlations of λ versus ν , γ versus ν , and λ versus γ with $\lambda = 1.6 \times 10^{-4}$; Eqs. (12) and (13) plotted, respectively.

fully developed waves, especially in the Great Lakes.

Correlations Between ϵ and ν

Deducing from nonlinear energy transfer calculations with an empirical calibration using JONSWAP data, Hasselmann *et al.* (1976) gave the following relationship between nondimensional energy ϵ and nondimensional peak frequency ν :

$$\epsilon = 5.3 \times 10^{-6} \nu^{-10/3} \tag{14}$$

They indicated that Eq. (14) is an equilibrium curve. As the waves grow, the equilibrium state migrates to the left along the curve toward lower frequencies and higher energies.

Toba (1978) proposed a similar relationship with a different coefficient and exponents

$$\epsilon = 7.1 \times 10^{-6} \nu^{-3} \tag{15}$$

Based on their own data, Mitsuyasu *et al.* (1980) found that

$$\epsilon = 6.84 \times 10^{-6} \nu^{-3} \tag{16}$$

Donelan (1977) used the following total energy E to represent the integral of JONSWAP spectrum

$$E = 0.294 \alpha g^2 (2\pi f_m)^{-4} \tag{17}$$

that leads to still another form with a different coefficient

$$\epsilon = 6.23 \times 10^{-6} \nu^{-10/3} \tag{18}$$

Hence we have exponents of either $-10/3$ or -3 with different coefficients by different authors: Figure 7 shows a comparison between the 880 data points and the correlations (14)–(17). We find that the data appear to be well correlated. This is one correlation that may have a universal application. However, the four proposed relations fit the data with varied degrees of closeness. Also the data points still cover a narrow region that does not allow a rational assessment of which exponent is better to use. By visual observations, Donelan's Eq. (17) seems to fit this data set better.

THE EPISODES

All the correlations we have presented are based on using the 880 data points with wave heights greater than 1 m. While some correlations are clearly poor or marginal, some are reasonably good. It is of interest to ask how these correlations fare during an actual episode. To this end, we selected four

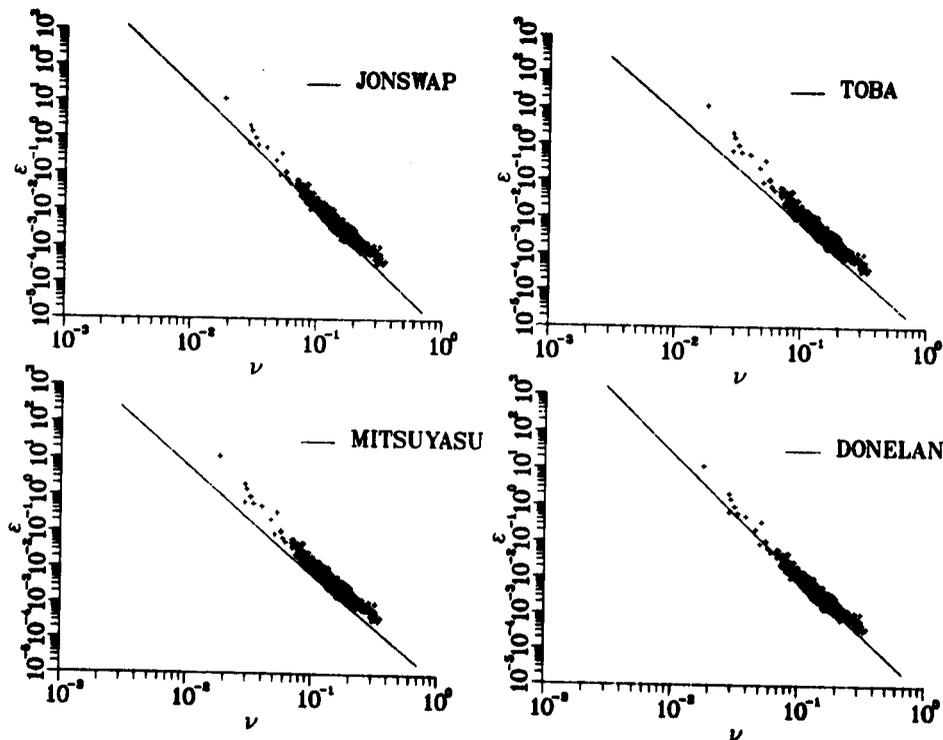


FIG. 7. Correlations of ϵ versus ν with Eqs. (14), (15), (16), and (17) plotted for JONSWAP, Toba, Mitsuyasu, and Donelan, respectively.

episodes from three different buoys. Each episode is 65 h long from early wave growth through continuous development to the peak of the storm. With the episodic data, we examined the correlations of ϵ versus ν , ϵ versus ξ , ν versus ξ , and SS versus $\epsilon\nu/\xi$, corresponding to Eqs. (14), (2), (1), and (3), respectively. In Figures 8–11, part a's present time series of wind direction, wind speed, and significant wave height during each episode, and part b's present the four correlations.

Episode 25–27 September, Figures 8a, b

This was recorded from the northern Lake Michigan buoy 45002. The wind speed follows a steadily increasing trend from 5 m s^{-1} to 15 m s^{-1} . The wind direction started from the south, which was long fetch, and changed to the west when the wind speed increased to over 10 m s^{-1} and the fetch distance reduced. The significant wave height was growing steadily, but only reached 3 m at the peak of the storm. The correlations show that the data points clustered around the established relations with a fair amount of scatter.

Episode 29 September–1 October, Figures 9a, b

This was recorded from the western Lake Superior buoy 45006. The wind speed during this episode were $6\text{--}9 \text{ m s}^{-1}$ for the most part, steadily from the east. The fetch distance was generally long. However, waves at buoy 45006 are influenced by the Keweenaw Peninsula located on the southern shore of Lake Superior. Without the peninsula, the fetch from the east at buoy 45006 could be much longer. This is an example of the difficulty in determining exact fetch even with an average of all the fetches with $\pm 15^\circ$ of the wind direction that yields an effective fetch to take care of the lake boundary effect. In this episode, it is clearly because of uniform wind speed and long fetch that the wave height has grown to nearly 5 m with wind speeds under 10 m s^{-1} . The correlations for this episode show some interesting results. The values for ξ were of the order 10^5 during the early part when the wave heights were low, and diminished to 10^4 during the last part when the waves started to decay and wind direction changed from east to north. The trend of ϵ versus ξ , ν versus ξ , and SS versus $\epsilon\nu/\xi$ correlations showed an entirely different orientation from the established relationships. If we did not have these *a priori* relationships and this was the only data set we had, we would most likely draw a different set of conclusions. The ϵ versus ν

correlation, on the other hand, was closely fitted with Eq. (14).

Episode of 16–18 October, Figures 10a, b

This is from the eastern Lake Superior buoy 45004, which recorded 7 m waves. We traced back 2 days to see the wind and wave history prior to the high waves. With winds blowing steadily from the south and wind speeds averaging about 8 m s^{-1} , wave heights were below 2 m. There was a brief decay early on 18 October, then the speed steadily increased from 3 m s^{-1} to 17.5 m s^{-1} , wind direction switched to northwest, and significant wave heights grew from 1 m to 7 m, all within 10 h (the total duration). The correlations during the growth part on 18 October follow the established straight-line relations quite closely. Prior to that, the points were somewhat scattered but still consistent with the lines.

Episode of 18–20 October, Figures 11a, b

This is the decaying part of the episode shown in Figure 10 (18 October). The wind speed diminished from 17.5 m s^{-1} to 5 m s^{-1} , followed by some brief increases; the wind direction was initially from the northwest, then from the southwest, and later from the north. The wave heights steadily decreased from 7 m to 1 m, and then mildly responded to the wind speed increases. The correlations are similar to those in Figure 10b with even less scattering. Thus the established correlations, while developed for growing waves, apply to the decaying waves as well.

These episodes represent a small glimpse of direct application of the parametric relationships to actual storm conditions. While they were selected for their general interest, they are by no means typical. We can only present a few scenes of what Phillips (1977) called a "dynamical kaleidoscope."

CONCLUSIONS

We have presented a detailed examination of various parametric correlations for their universality using a large number of data points. We found many correlations where data points are scattered widely. Some, where points are clustered around a distinctive region rather than a straight line, may be considered quasi-universal correlations. The variations within the region can be up to one order of magnitude or more. Applying one of these rela-

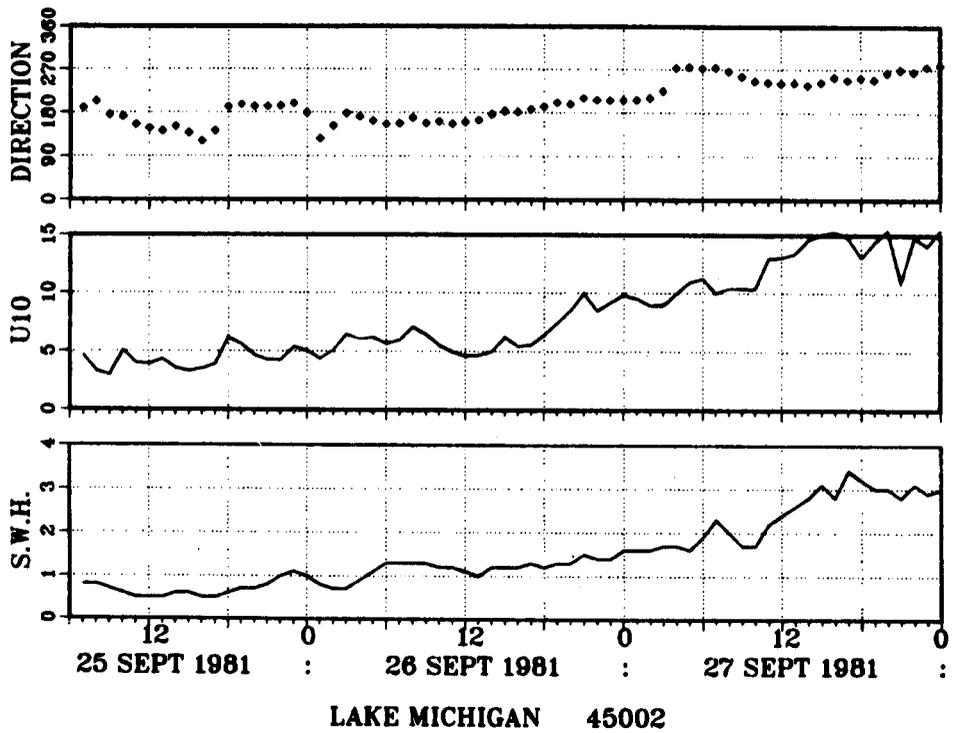


FIG. 8a. Time series for episode 25-27 September 1981.

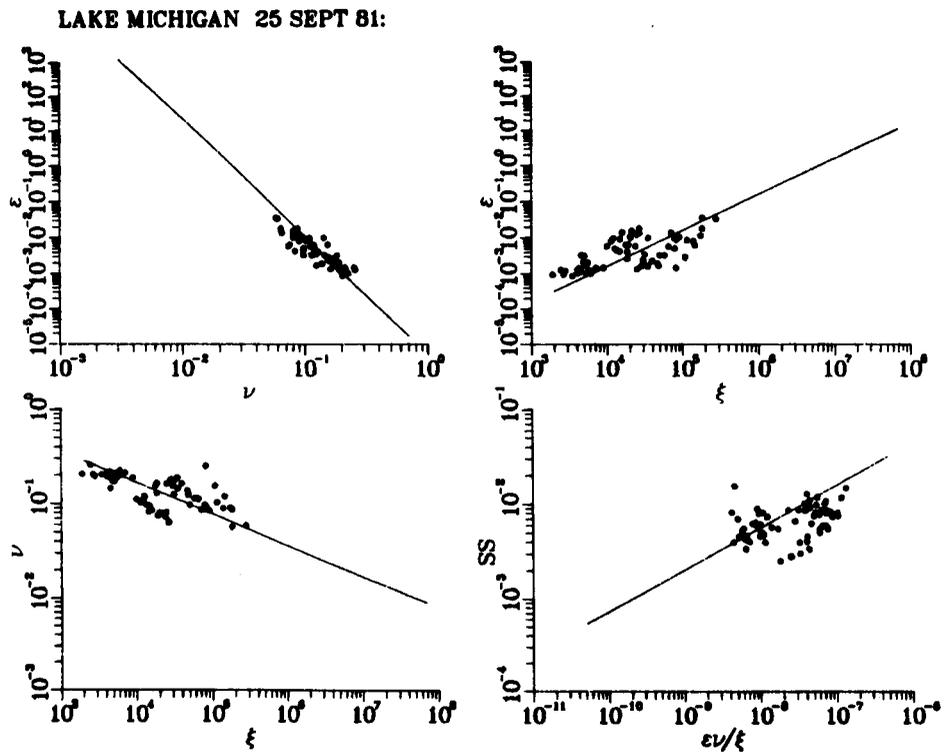


FIG. 8b. Correlation of ϵ versus ν , ϵ versus ξ , ν versus ξ , and SS versus $\epsilon\nu/\xi$ with Eqs. (14), (2), (1), and (3) plotted, respectively. Data from the episode shown in a.

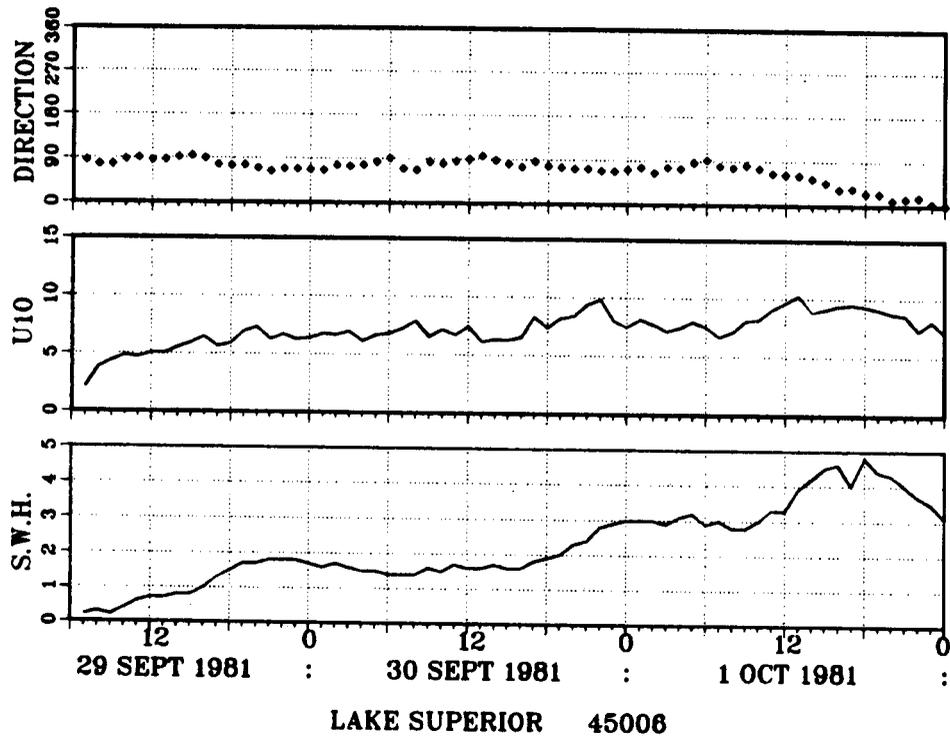


FIG. 9a. Time series for episode 29 September-1 October 1981.

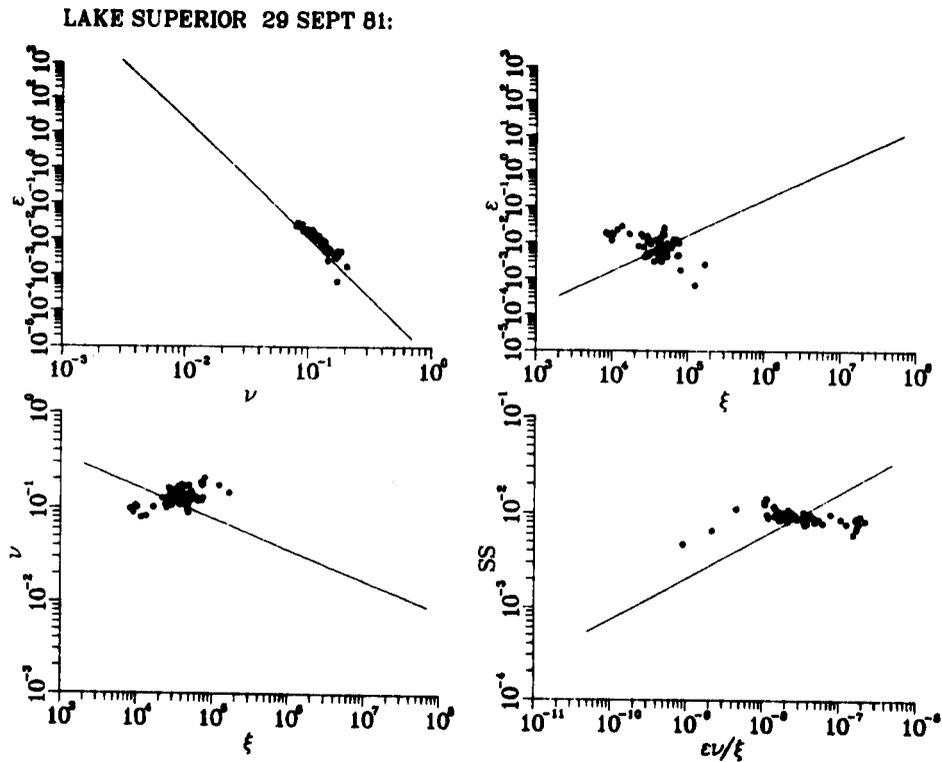


FIG. 9b. Same as Figure 8b.

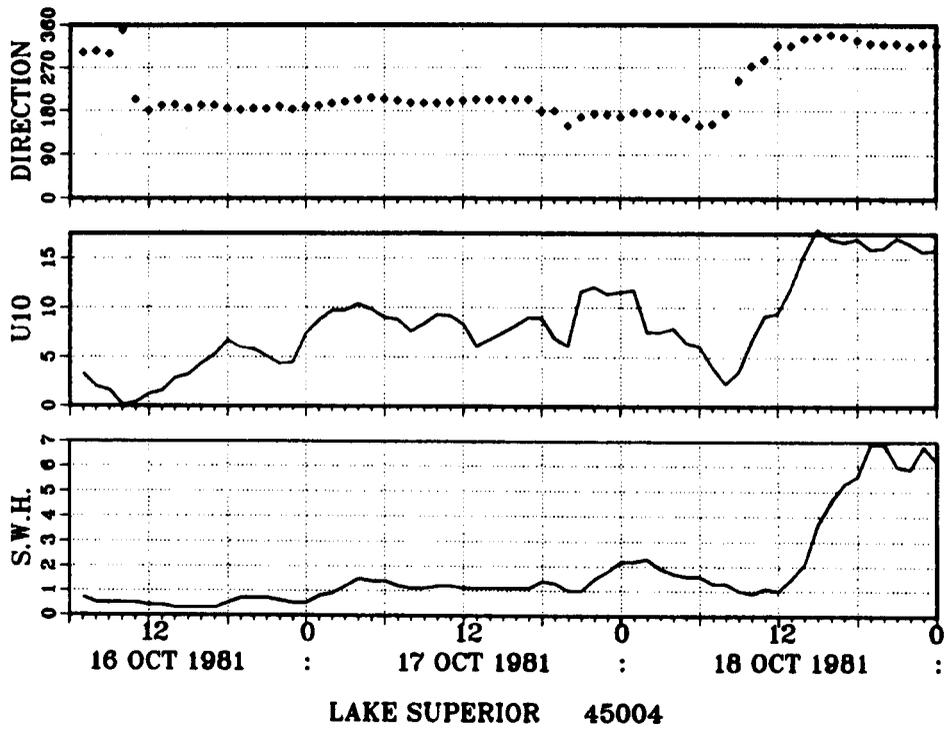


FIG. 10a. Time series for episode 16-18 October 1981.

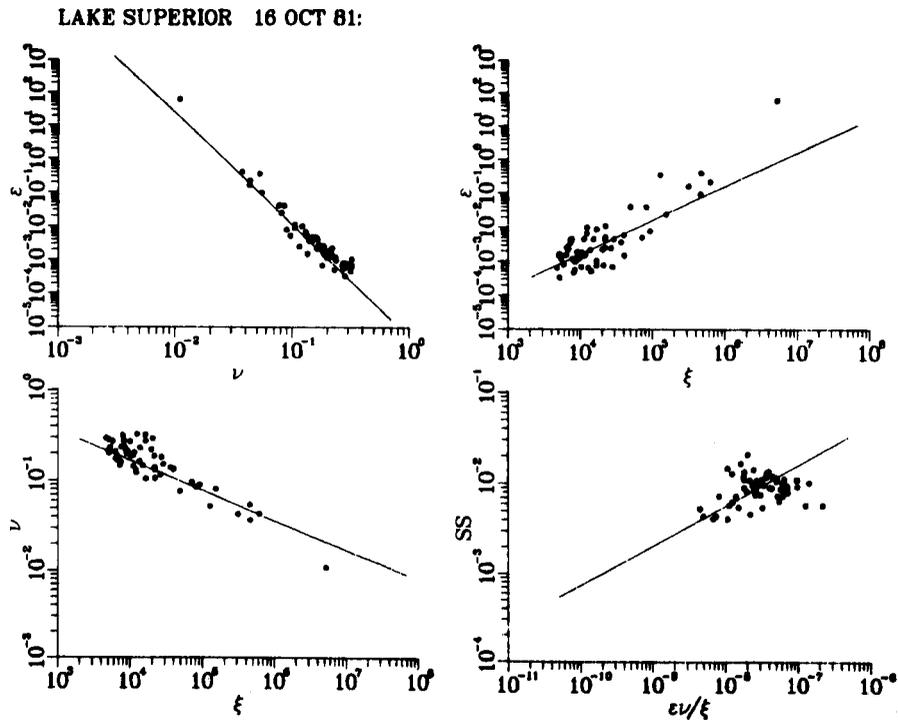


FIG. 10b. Same as Figure 8b.

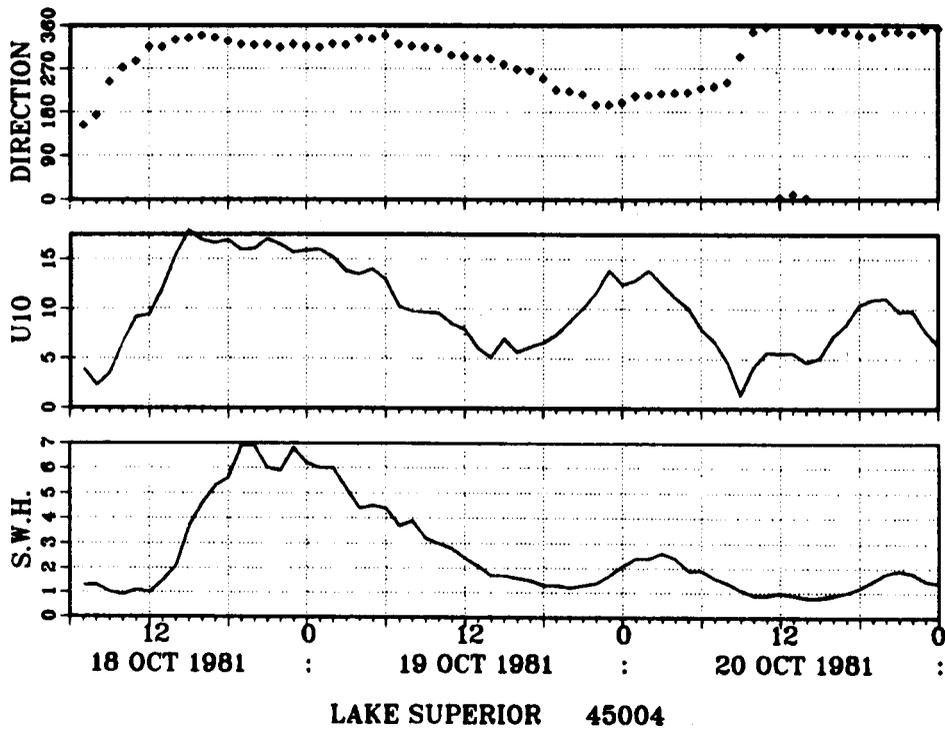


FIG. 11a. Time series for episode 18-20 October 1981.

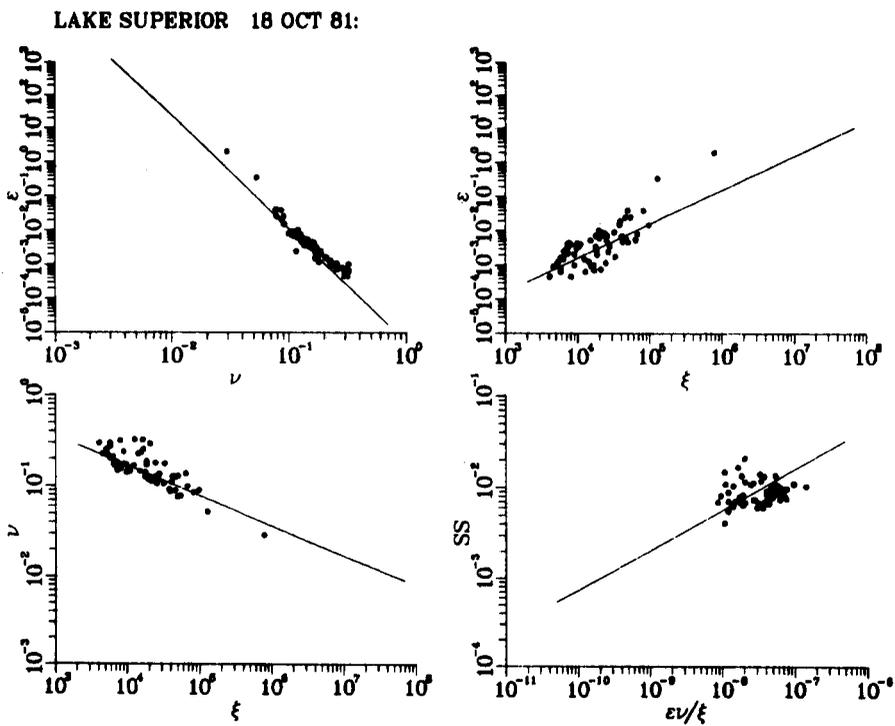


FIG. 11b. Same as Figure 8b.

tions in practice can be quite accurate at times and erroneous at other times.

We were not able to find clear discernible seasonal or atmospheric stability effects to sort the correlations. But we found that distinctive correlations are produced under different wind speeds. Clearly, when fetches do not vary significantly, wind speed is the only major factor that controls the correlations.

In this study all normalizations were made with U_{10} , because it is more readily available and has been used for sealing in many previous studies. However, normalizing the parameters with frictional velocity U^* instead of U_{10} (Liu 1984) indicates that the correlations with U^* normalization are substantially similar to the U_{10} normalization presented here.

Among the correlations, the one that consistently exhibits a universal behavior is that of nondimensional energy ϵ versus nondimensional peak frequency ν . This confirms the work of Toba (1978) and Mitsuyasu *et al.* (1980). Application of this correlation in furthering model development has been fruitful. Applying the ϵ versus ν power law (15), Toba and his colleagues (Kawai *et al.* 1979) developed a successful wave prediction model. Using Donelan's correlation (17) for parameterization, Schwab *et al.* (1984) and Liu *et al.* (1984) also developed a simple wave prediction model with clearly tractable empiricism that produced quite satisfactory results.

Until truly universal relations are found and if preciseness is not specifically required, the currently available correlations can be used with limited success. However, the task of searching for universal parametric correlations for wind waves continues to confront oceanographers.

ACKNOWLEDGMENT

I thank Drs. L. H. Holthuijsen and C. Bishop for their constructive reviews. GLERL Contribution No. 425.

REFERENCES

- Businger, J. A., Wyngaard, J. C., Izumi, Y., and Bradley, E. F. 1971. Flux-profile measurements in the atmospheric surface layer. *J. Atmos. Sci.* 28:181-189.
- Charnock, H. 1955. Wind stress on a water surface. *Quart. J. Roy. Meteorol. Soc.* 81:639.
- Donelan, M. A. 1977. *A simple numerical model for wave and wind stress prediction*. Report, National Water Research Institute, Burlington, Ont., Canada.
- Hasselmann, K. 1977. On the spectral energy balance and numerical prediction of ocean waves. In *Turbulent Fluxes Through the Sea Surface, Wave Dynamics and Prediction*. pp. 531-543. A. Favre and K. Hasselmann, Eds., New York: Plenum Press.
- _____, Barnett, T. P., Bouws, E., Carlson, H., Cartwright, D. E., Enke, K., Ewing, J. A., Gienapp, H., Hasselmann, D. E., Kruseman, P., Merrburg, A., Muller, P., Olbers, D. J., Richter, K., Sell, W., and Walden, H. 1973. Measurements of wind-wave growth and swell decay during the Joint North Sea Wave Project (JONSWAP). *Deut. Hydrogr. Z.* A12.
- _____, Ross, D. B., Muller, P., and Sell, W. 1976. A parametric wave prediction model. *J. Phys. Oceanogr.* 6:200-228.
- Huang, N. E., Long, S. R., and Bliven, L. F. 1981. On the importance of the significant slope in empirical wind wave studies. *J. Phys. Oceanogr.* 11:569-573.
- Kawai, S., Joseph, P. S., and Toba, Y. 1979. Prediction of ocean waves based on the single-parameter growth equation of wind waves. *J. Oceanogr. Soc. Japan* 35:151-167.
- Kitaigorodskii, S. A. 1961. Application of the theory of similarity to the analysis of wind-generated wave motion as a stochastic process. *Bull. Acad. Nauk SSSR Geophys. Ser.* 1:105-117.
- Liu, P. C. 1984. In search of universal parametric correlations for wind waves. Paper presented at the Symposium on Wave Breaking, Turbulent Mixing and Radio Probing of the Ocean Surface, July 19-25, 1984, Tohoku University, Sendai, Japan.
- _____, and Ross, D. B. 1980. Airborne measurements of wave growth for stable and unstable atmospheres in Lake Michigan. *J. Phys. Oceanogr.* 10:1842-1853.
- _____, Schwab, D. J., and Bennett, J. R. 1984. Comparison of a two-dimensional wave prediction model with synoptic measurements in Lake Michigan. *J. Phys. Oceanogr.* 14:1514-1518.
- Mitsuyasu, H., Tasai, F., Suhara, T., Mizuno, S., Ohkusu, M., Honda, T., and Rikiishi, K. 1980. Observation of the power spectrum of ocean waves using a cloverleaf buoy. *J. Phys. Oceanogr.* 10:286-296.
- Phillips, O. M. 1958. The equilibrium range in the spectrum of wind-generated waves. *J. Fluid Mech.* 4:426-434.
- _____. 1977. *The Dynamics of the Upper Ocean*, 2nd Ed., Cambridge, England: Cambridge University Press.
- Schwab, D. J., Bennett, J. R., Liu, P. C., and Donelan, M. A. 1984. Application of a simple numerical wave prediction model to Lake Erie. *J. Geophys. Res.* 89(C3):3586-3592.
- Steele, K., and Johnson, Jr., A. 1977. Data buoy wave measurements. In *Ocean Wave Climate*, pp. 301-316. M. D. Earle and A. Malahoff, Eds. New York: Plenum Press.

Sverdrup, H. Y., and Munk, W. 1947. *Wind, sea and swell: Theory of relations for forecasting*. Publ. No. 601, U.S. Hydrographic Office, Washington, D.C.

Toba, Y. 1978. Stochastic form of the growth of wind waves in a single-parameter representation with physical implications. *J. Phys. Oceanogr.* 8:494-507.