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OBSERVATION OF EKMAN VEERING AT THE BOTTOM OF LAKE MICHIGAN[1]

James H. Saylor and Gerald S. Miller
National Oceanic and Atmospheric Administration
Great Lakes Environmental Research Laboratory
2205 Commonwealth Blvd.
Ann Arbor, Michigan 48105

ABSTRACT. An experiment in 100 m of water off the east coast of Lake Michigan during 1984 provided continuous current velocity recordings at four levels located from 1 m to 9 m above the lake bottom and also at 50 m. Near-inertial-period current oscillations were prominent and superimposed on longer-period velocity variations. Current speed at 1 m elevation varied from less than the threshold speed of the Savonious rotor sensor to 13 cm s^{-1}. Profiles of current speed were close to logarithmic during many episodes of measurable currents over a 4-month-long recording interval. Over 50% of hourly-averaged profiles were in the nearly-logarithmic category ($R > 0.987$) during the month of September 1984. Veering averaging 11° was observed between low-pass-filtered current velocities measured at elevations of 1 m and at 9 m. Boundary layer theories developed from ocean studies are surveyed and the Lake Michigan data are examined from that perspective.
ADDITIONAL INDEX WORDS: Water currents, current meters, lake circulation, boundary layers.

INTRODUCTION

Bottom boundary layers in the atmosphere and in the ocean have been much studied since Ekman first showed that in the northern hemisphere for stationary, horizontally-uniform flow with constant eddy viscosity, the velocity vector must veer counterclockwise looking downward. Using similarity arguments, theoretical studies of turbulent Ekman layers (e.g., Csanady 1967, Gill 1968) have shown that the boundary layer can be divided into an inner and outer region. The inner region acts as a usual nonrotating boundary layer on a flat plate. The current velocity vectors do not rotate with depth because the Coriolis effects are negligible and the stress is nearly constant and equal to the wall shear stress τ_o (equal to ρu_*^2 where ρ is the density and u_* is the friction velocity). An Ekman-layer-like structure exists in the outer region in that the Coriolis forces balance the Reynolds stress terms and the current vectors rotate with depth. The velocity profiles overlap and match in their common reach.

Observations of bottom Ekman layers in the oceans have been reported by Dickey and VanLeer (1984), by Kundu (1976), and by others (e.g., reviews by Nowell 1983 and Grant and Madsen 1986). Dickey and VanLeer's observations were made in a high velocity environment (free stream flow above the bottom boundary layer of about 30 cm s^{-1}) on the Peruvian continental shelf in 100 m water depth. Kundu's observations near the Oregon coast were in 100-m and 200-m depths in low flow velocities similar to those observed offshore during summer in the Great Lakes. He found Ekman veering of about 6° between current meters placed 5 m and 20 m above the ocean floor at the 100-m depth site. Because veering is potentially of much significance to the transport of resuspended sediments within the deep basins of the Great Lakes, we report here some results of lake bottom current studies in southern Lake Michigan that give evidence of Ekman layer current structure. Studies of Ekman layers in the ocean bottom boundary layer are noted and the Lake Michigan data are examined from that theoretical framework.

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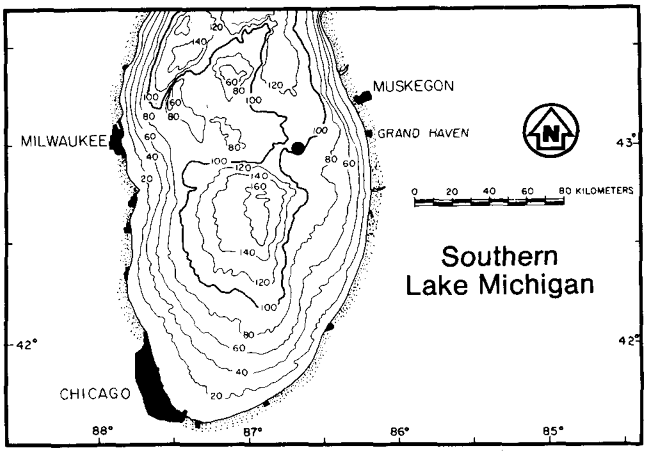

FIG. 1. Bathymetry of the southern basin of Lake Michigan and location of the current measurement site.

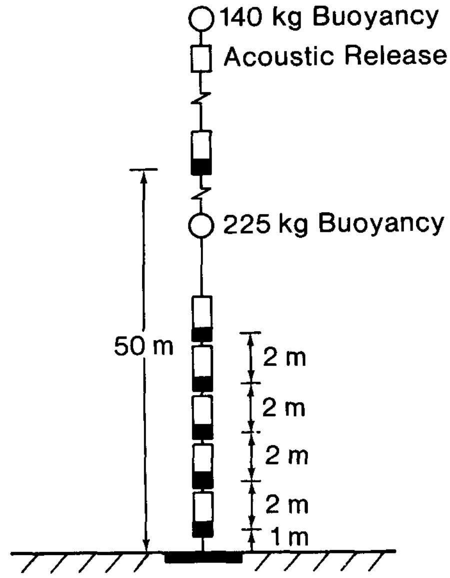

FIG. 2. Configuration of the current meter mooring.

OBSERVATIONS

The current meter data were collected at a site about 30 km west-southwest of Grand Haven, Michigan (Fig. 1). The location was on a broad plateau of uniform depth about 100 m, so that either offshore or alongshore depth gradients were very slight. The bottom sediments were a mixture of silt and clay-sized materials with virtually no relief.

Currents were measured with E G & G vector averaging current meters moored as shown in Figure 2. The current speed sensors were placed at heights of 1, 3, 5, 7, 9, and 50 m above the lake floor. The current velocity is continuously sampled and vector averaged in this type of meter; the averaged velocity vector was recorded at 15-minute intervals. The 15-minute observations were averaged to give hourly data sets from each meter. For computing current speed profiles the hourly data were used directly.

The current meters were deployed from mid-June through mid-October 1984. Flow velocities near the bottom were very low throughout June and July with many zero speed recordings (the threshold speed of the meters is about 2 cm s^{-1}). This period will not be considered here. We will discuss only the data collected during August, September, and the first half of October, an interval about 75 days in length.

CURRENT SPEED PROFILES

Ekman showed that for the case of constant eddy viscosity, current speed increased exponentially above the seafloor until the flow matched the overlying gradient current (Sverdrup *et al.* 1942). Although the assumption of constant eddy viscosity is by first principles not reasonable, similarity theory of modern planetary boundary-layer analysis (Grant and Madsen 1986) still predicts logarithmic current speed profiles. We often observed speed profiles in Lake Michigan that were close to logarithmic.

The time-averaged, near-bottom velocity profile in a turbulent boundary layer over a topographically simple bottom is of the form

$$u = \frac{u_*}{k} \ln \frac{z}{z_0}, \tag{1}$$

where u is the mean horizontal velocity, k is Von Karman's constant, z the height above the bottom, z_0 the bottom roughness and u_* is the shear velocity associated with the mean flow ($u_* = (|\tau_0|\rho^{-1})^{1/2}$). Measurements of hourly-averaged current speed during a 7-hour-long interval on 7 September 1984 are shown in Figure 3. Defective rotor bearings in the current meter 5 m above the bottom led to speed underestimates at that level. The interval is representative of the best logarithmic profiles observed. The confidence band on the friction velocity estimate depends on the regression coefficient, on the number of current meters, and on their location in the vertical. Gross and Nowell (1983) used the relation

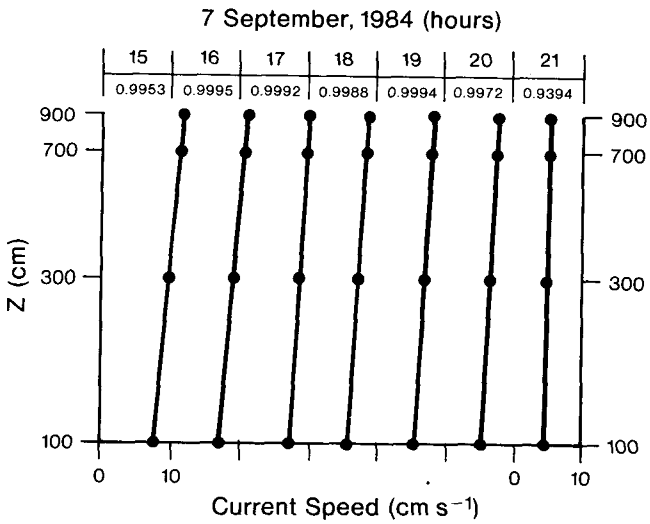

FIG. 3. Profiles of current speed measured on 7 September 1984. Scale of current speed is 0–10 cm s^{-1} for each profile. Regression coefficients are at the top of each profile.

$$u_*' (1 - e) \leq u_*' \leq u_*' (1 + e), \qquad (2)$$

where u_*' is the estimate of u_* and

$$e = (t_{\alpha/2,n-2}) \left[\frac{1}{n-2}\left(\frac{1-R^2}{R^2}\right)\right]^{1/2}, \qquad (3)$$

where t is the Students t distribution for the (1-α) confidence interval with n-2 degrees of freedom, n is the number of current meters, and R is the regression coefficient. We are 95% confident that u_* is within $\pm$ 50% of its actual value if $R > 0.987$, or that u_* is within $\pm$ 25% of its actual value if $R > 0.997$ for four current meters.

Bottom stress and roughness can be estimated from the measured current speed profiles. Writing (1) as

$$\log z = \frac{k}{2.3u_*} u + \log z_o \qquad (4)$$

gives the equation for a straight line with a plot of log z versus u. The slope of the line is proportional to the reciprocal of the friction (shear) velocity and the z intercept is the logarithm of the roughness length. The profiles must be logarithmic for these values to have any meaning, but there have been no general agreements on the regression coefficients or standard errors required to use the method. Grant *et al.* (1984) imposed the very strict criterion of $R > 0.997$ for measurements in the inner logarithmic layer in order to determine u_* to within $\pm$ 25%. Other criteria used have generally been much less stringent.

In the Ekman layer, there have been few comparisons of velocity profiles with logarithmic expectations. The regression coefficients associated with the profiles during a 7-hour episode (Fig. 3) meet the very strict criteria demanded by Grant *et al.* (1984) in their profiles nearer to the bottom. We find that the bottom current speed profiles in Lake Michigan often exceed this strict criterion. For example, when current speed at 1 m above the bottom was at least 4 cm s^{-1}, 26% of profiles measured during the month of September had regression coefficients greater than 0.997, 52% exceeded 0.987, and fully 78% exceeded 0.967 (48% of the September currents exceeded 4 cm s^{-1} at 1 m above the bottom). Grant *et al.* found 30% of their profiles had regression coefficients exceeding 0.997 in a short (15-hour-long) episode of high quality data recordings in the inner logarithmic layer.

If we use a larger number of the hourly-averaged observations, the logarithmic quality of the data is poorer. Speed was greater than 2 cm s^{-1} in 92% of the September recordings; 21% of these less restricted profiles had regression coefficients larger than 0.997. Low speed therefore makes the detection of logarithmic profiles more difficult. This must be partly due to the fact that the Savonius-rotor sensors we used are inherently less accurate near their threshold of measurement.

Another factor of interest is the thickness of the Ekman layer itself. Figure 4 shows current speed recordings close to the bottom and at mid-depth in the water column (50 m above the bottom) during an interval in September. At the start of the interval current speed at 9 m and 50 m were similar, with the speed at 9 m being 80% to 90% of that recorded at mid-depth. When these conditions prevailed, throughout intervals of significant currents, the best logarithmic profiles occurred. When speed near the bottom deviated significantly from what we consider as the driving force, i.e., the gradient current above the boundary layer, the logarithmic quality of the measurements was poorer. A thermocline at depths of 20 to 30 m persisted throughout the measurement period, but temperature differences between the 1 and 9 m levels were less than 0.15C° and stratification was minimal.

Factors governing the thickness of the bottom Ekman layer were considered by Weatherly *et al.* (1980). They quote an often-used approximate value for this thickness as

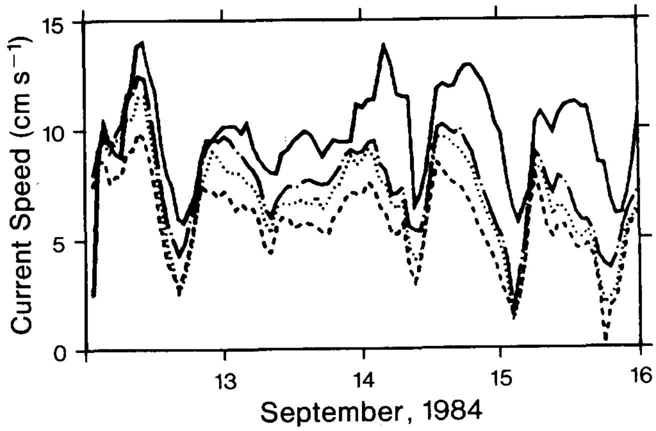

FIG. 4. Observed current speeds at 50, 9, 7, and 3 m (high to low speeds, respectively) above the bottom during a 4-day-long interval in September 1984.

$$h = 0.4\, u_*f^{-1}, \tag{5}$$

where u_* is the friction velocity and f is the Coriolis parameter. An average value for u_* from our profile measurements was 0.5 cm s^{-1}, giving a layer thickness of 20 m. The measurements certainly indicated that the thickness exceeded 9 m, since the current speed at 9 m was at times just a small percentage of the current speed at mid-depth (Fig. 4). Smith (1977) suggested that a reasonable estimate of the thickness of the inner logarithmic layer is given by $0.03\, u_*f^{-1}$, of order 1 m here.

Munk *et al.* (1970) considered effects of rotational flows on boundary layer thickness and determined the Ekman layer thickness, δ, for steady (δ_s), clockwise rotating (δ_+), and counterclockwise rotating (δ_-) flow to be

$$\begin{aligned} \delta_s &= (2k/|f|)^{1/2}, \\ \delta_+ &= (2k/|\omega - f|)^{1/2}, \\ \delta_- &= (2k/|\omega + f|)^{1/2}, \end{aligned} \tag{6}$$

where k is the eddy viscosity (considered constant) and ω is the rotation frequency. Weatherly *et al.* (1980) used these relations to discuss the thickness of the bottom layer associated with tidal currents. They found significant effects if the amplitude of the clockwise rotating component was similar in magnitude to the steady current. In Lake Michigan near-inertial-period currents, with ω slightly larger than f, are omnipresent features of the stratified lake (Mortimer 1980). Figure 5 shows their prominence in a spectrum of kinetic energy at the current meter 9 m above the bottom. The current rotation is clockwise and (6) shows that the boundary layer can be very thick and may in fact encompass the entire water column during intervals when the near-inertial currents are the principal deep flows. A very thick boundary layer would be characterized by large differences between mid-column and deep-layer flows, as Figure 4 illustrates.

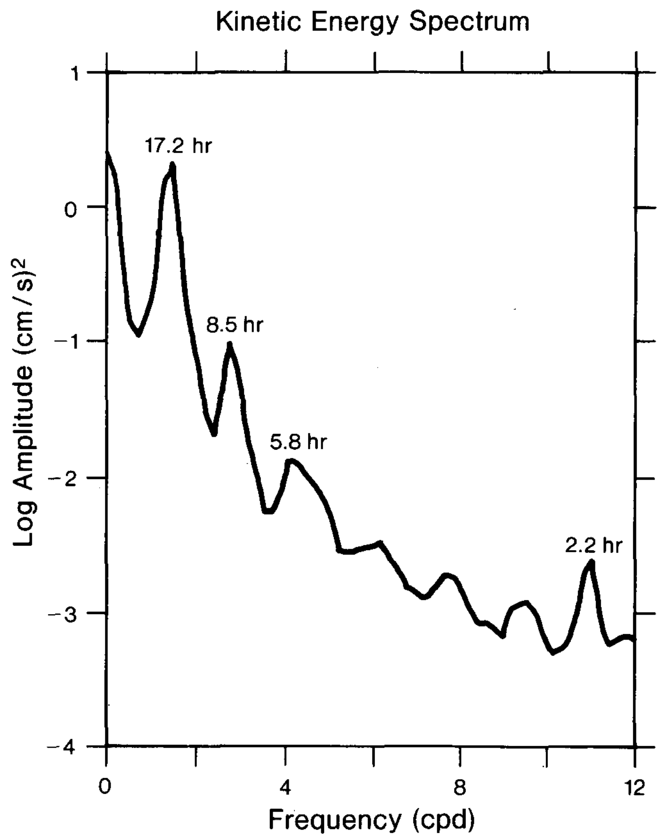

FIG. 5. Kinetic energy spectrum computed from the current velocity components recorded at 9 m above the bottom during 1984.

Internal waves may distort the velocity profiles resulting in erroneous friction velocity and roughness length values when time averages are small compared to the inertial period (Grant 1982). The constant presence of near-inertial currents in this data set precludes a comparison to profiles during times of steady flow.

CURRENT VEERING

For detection of stationary Ekman-like characteristics of the bottom boundary layer it is necessary to examine the frequencies of range smaller than the inertial frequency (Kundu 1976). A low-pass filter having a half-power point of 40 hours was applied to the mean hourly current velocity time series to eliminate frequencies > 0.6 cpd. One method of determining the angular displacement of the current between two levels is to average arithmetically the direction deviation from the low-frequency time series

$$\alpha_{av} = <\alpha(t)> = <\tan^{-1}\frac{v_2}{u_2} - \tan^{-1}\frac{v_1}{u_1}>$$

$$= <\tan^{-1}\frac{u_1u_2 - v_1v_2}{u_1u_2 + v_1v_2}>, \quad (7)$$

where u and v are the east and north components of the velocity at each level (level 2 closer to the bottom than level 1), the angle α is positive if the vector at level 2 lies counterclockwise of the vector at level 1, and the angle braces denote a time average. Kundu (1976) pointed out that this average gives equal weight to each hourly observation, although we observed earlier that the bottom flow was less well structured at low current speeds. Erratic veering at low current speeds (especially during June and July) could easily obscure a persistent low angle variation between levels.

Kundu (1976) introduced a method that weights the averaging process according to the magnitude of the individual velocity vectors in order to avoid the low speed problem just discussed. He used the phase angle of the complex correlation coefficient between two vector series. If $w(t) = u(t) + iv(t)$, where $i = \sqrt{-1}$, is the complex representation of the velocity vector, the complex correlation coefficient is defined as

$$\gamma = \frac{<W_1^*(t)\, W_2(t)>}{<W_1^*(t)\, W_1(t)>^{1/2} <W_2^*(t)\, W_2(t)>^{1/2}}, \quad (8)$$

where the asterisk denotes the complex conjugate. The coefficient, γ, then gives the overall measure of correlation between the two vector series whose phase angle is the average counterclockwise angle of veering of the second vector series with respect to the first. In terms of the u and v components

$$\gamma = \frac{<u_1u_2 + v_1v_2> + i <u_1v_2 - u_2v_1>}{<u_1^2 + v_1^2>^{1/2} <u_2^2 + v_2^2>^{1/2}}, \quad (9)$$

and the average veering is

$$\alpha_{av} = \tan^{-1}\frac{<u_1v_2 - u_2v_1>}{<u_1u_2 + v_1v_2>}. \quad (10)$$

We computed the veering angles from both (7) and (10) for the pair of current meters that were 1 and 9 m above the bottom. Results are shown in Table 1 for 7-day-long intervals starting on 30 July 1984. Estimates by the two methods are not very different, indicating that the veering is steady over 7-day intervals. Veering between the 1- and 3-m levels is also listed in the table. Values of the correlation coefficient are high and comparable with those Kundu observed off the Oregon coast. If the analysis were constrained to the intervals of the highest steady current speeds, i.e., September and October, the correlation coefficient is about 0.98.

Table 1 reveals consistent counterclockwise turning of the velocity vector looking down through the water column toward the lake bottom. Comparisons of the 7- and 9-m levels and of the 3- and 7-m levels likewise show similar velocity rotation between each pair of meters. The table indicates the veering is most rapid close to the lake bottom.

VEERING OF INERTIAL CURRENTS

Errors are always possible in current measurements of this type and separate estimation of potential errors is helpful. Figure 5 showed a strong inertial period component of the currents. It occurred at all measurement levels. Cross spectra between the four lowest current velocity recordings revealed high coherence only at very low frequency and at the inertial frequency where the coherence is near unity (Fig. 6). Estimates of the phase lag between current meters could therefore reveal serious direction errors. The spectral computations revealed less than 1° phase shift between current meters 1 and 3 m off the bottom, and less than 1° difference between the 7- and 9-m levels. This means that the direction measurements and interval timing were precise. A 15-minute timing error would cause a 5° phase shift at the inertial period.

Between the upper and lower pairs of meters we observed a phase shift averaging 4°, with current directions at the bottom pair rotated counterclockwise from those above. It is not very likely that two adjacent meters would experience the same timing error. A more credible explanation is that even at the inertial period we observe a small counterclockwise veering of the velocity vector as the bottom is approached.

TABLE 1. Ekman veering averaged over 7-day-long intervals during 1984. Columns labeled (7) and (10) give averages determined from equations (7) and (10), respectively, for each meter pair.

1984 JULIAN DAYS	ANGLE OF VEERING BETWEEN 3 m & 1 m (7)	3 m & 1 m (10)	9 m & 1 m (7)	9 m & 1 m (10)	VECTOR SERIES CORRELATION COEFFICIENT 3 m & 1 m	9 m & 1 m
210-216	7.2°	8.5°	2.7°	9.4°	0.950	0.962
217-223	10.2°	9.2°	9.5°	12.0°	0.948	0.963
224-230	4.5°	10.7°	12.9°	15.9°	0.951	0.906
231-237	9.3°	11.0°	14.2°	14.6°	0.965	0.962
238-244	8.5°	8.3°	11.6°	13.6°	0.946	0.966
245-251	4.6°	6.2°	7.2°	8.9°	0.992	0.983
252-258	3.7°	3.8°	5.4°	5.7°	0.996	0.992
259-265	5.5°	6.6°	10.8°	11.8°	0.988	0.975
266-272	12.5°	11.1°	15.3°	15.2°	0.974	0.957
273-279	5.3°	6.3°	7.0°	8.3°	0.970	0.980
280-286	3.8°	7.0°	3.0°	7.6°	0.973	0.979
Average	6.8°	8.8°	9.1°	11.2°		

DISCUSSION OF RESULTS

Measurements of current velocity within 9 m of the bottom at a site in 100 m water depth in Lake Michigan provided a large percentage of nearly logarithmic speed profiles. The highest quality measurements permit accurate estimation of u_* in the lower part of the Ekman layer as well as estimates of roughness length, bottom stress, and drag coefficients. Using the most logarithmic September profiles (R > 0.997 and speed at 1 m > 4 cm s^{-1}) we find that $\bar{u}_* = 0.6$ cm s^{-1} with a range of 0.28 to 0.88 cm s^{-1} and $\bar{z}_0 = 2.7$ cm with a range of 0.04 to 10.5 cm. Using the quadratic stress relation $\tau_0 = C_d \rho u^2$, we can compute a drag coefficient at 1 m elevation using the observed u_{100} and u_* values. The mean of C_{100} was 17×10^{-3}, a value 5 to 10 times larger than normally associated with logarithmic profile measurements in the inner logarithmic layer (c.f., Sternberg 1968). Bottom stress would be an unusually large percentage of the surface stress from these observations. Stresses and profile data meaningful to resuspension and sediment transport processes require data collected much closer to the bottom itself (Chriss and Caldwell 1982).

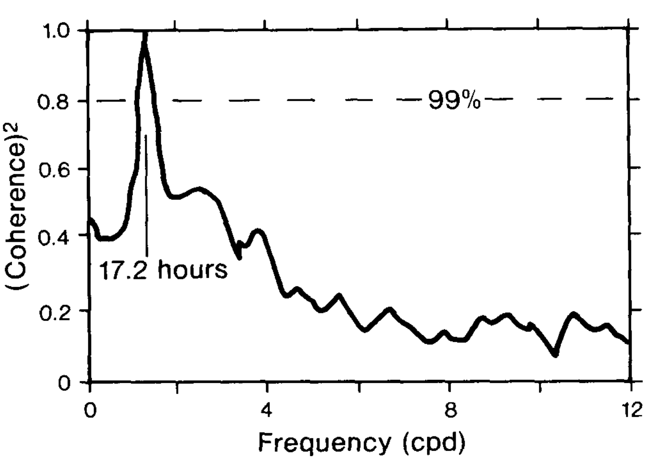

FIG. 6. Coherence between current meter velocity recordings at 1 and 3 m above the bottom. The 99% confidence level is shown by the dashed line.

All of the values are more in agreement with Sternberg's findings if we confine our attention to profiles observed during a high current speed episode. With the speed at 1 m > 10 cm s^{-1}, $\bar{u}_* = 0.76$ cm s^{-1} and $\bar{z}_0 = 0.31$ cm with a range of 0.11 to 0.74 cm. The mean drag coefficient was 4.5×10^{-3} with a range of 3.1 to 6.6×10^{-3}. Because the high speed episode was driven by an impulse of strong wind stress, the bottom stress computed from this data is more reasonably 10 to 20% of the surface wind stress. As the inner logarithmic layer is thicker with stronger current flow, the 1- and 3-m levels may well have been in that layer during the episode.

Consistent counterclockwise veering of the velocity vector between each pair of current meters, from 9 m elevation to 1 m, yields convincing evidence of Ekman layer flow at the bottom of Lake Michigan. Significant errors in the current meter direction measurements were ruled out by cross-spectral computations between meters. Low-pass-filtering of the hourly data was done because only the low frequency motions with periods longer than the inertial period were expected to resemble the steady-state Ekman solution. Obviously small timing errors between current meters could give false indication of veering if the large amplitude inertial motions were retained. However, timing errors were found to be negligible; and in fact we noted a slight counterclockwise rotation of the velocity vector toward bottom even at the inertial period.

Boundary-layer-type logarithmic current speed profiles and Ekman veering have not been previously reported in the Great Lakes. The structure of the bottom flow in the deep lake basins is an important determinant of sediment distribution and deposition; persistent gyres either concentrate or disperse the materials. It is noteworthy that the bottom currents behave so similarly to those observed in the oceans, despite the higher intensity of near-inertial-period oscillations in Lake Michigan.

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