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Front Cover: The cover illustration is a three dimensional global view of historically measured July precipitation (see paper 4.6, pg. 128). The view shown here is looking north from the South Pole and the dateline with the continents colored to provide geographical orientation. A series of twelve monthly images, all with the same scaling and viewpoint, is easily generated on the Macintosh and played back as a movie showing the seasonal cycle. These graphics were generated by Stanley Grotch of the Program for Climate Model Diagnosis and Intercomparison at the Lawrence Livermore National Laboratory, Livermore, California.

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USE OF CART FOR DIAGNOSTIC AND PREDICTION PROBLEMS IN THE ATMOSPHERIC SCIENCES

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1. INTRODUCTION

Tree-based statistical models are a recent development in statistics which have been applied to diagnostic and prediction problems in widely diverse fields of endeavor, but are as yet not well known in the atmospheric sciences. These are an alternative to linear and additive models for regression problems and to linear logistic and additive logistic models for classification problems. Much of the pioneering work in tree-based statistical model development was done by Breiman et al (1984) in their development of CART, which stands for Classification and Regression Trees. CART is a tree-based non-parametric statistical procedure for application to classification and regression problems. Its authors find that error rates of CART solutions are nearly always as low or lower than solutions by parametric procedures such as linear regression, logistic regression, and discriminant analysis, and are significantly lower for problems involving complex predictands and many predictors. The software for running tree-based models can be obtained from its developers, and more recently, has been included in the "S-Plus" statistical software package (Chambers and Hastie, 1992).

From a data base of predictand cases and accompanying predictors CART will establish decision trees that either classify a categorical predictand or are a regression fit of the predictand. A decision tree consists of a tree-like structure of binary decision rules. At each decision point (node) a case will branch either to the left or to the right based on a test against a threshold predictor value, and will continue branching in subsequent nodes until a final point (terminal node) is reached. CART uses a cost-complexity measure based on error rate and tree complexity to determine how many nodes it will allow. It uses this measure to search for the optimal tree, i.e. the tree which would give the least error when used with independent data. The error is calculated with a test data set held in reserve for data sets larger than about 1000 cases, or by cross-validation for smaller data sets. Both categorical and continuous predictors are allowed, and linear combinations of predictors can be tried. Several options for determining node splitting rules are allowed. Predictors are ranked in an ad-hoc manner according to their importance in establishing the trees. The decision trees found in the work reported here have proved to be appealing to users because they are easy to understand and work with, and the decision rules are nearly always found to make physical sense.

Use of CART in a classification application was reported in Burrows (1991). In that paper a tree-based statistical model was developed for mesoscale prediction of snowfall from lake-effect snowsqualls. The model has been implemented at the Ontario Weather Center. The forecasts have been found by the operational forecasters to be accurate and useful, and the guidance is now used routinely in the office.

The application of CART reported in this paper is the development of a tree-based statistical regression model for prediction of ice-cover on the Great Lakes. Observed spatially-averaged ice-cover for the winter months December-April of 1965-1979 for six large basins defined in Assel (1990) for Lakes Superior and Erie were used as the learning data to develop tree-based regression prediction models for each basin. Eight potential predictors relevant to this problem were used. Three tree-based ice-cover statistical prediction models were developed for each basin and used with some simple selection rules to produce a final model for daily ice-cover prediction in each basin for early November to late May. The models have approximately 1/3 to 2/3 the RMS error of Assel's (1990) freezing-degree-day (FDD) model in fitting the observed data. When applied to independent data they are expected to be capable of predicting percentage ice-cover for Lake Superior with about 10-20% error and for Lake Erie with about 15-20% error. The new models should be suitable for prediction of daily to weekly spatially-averaged ice-cover in numerical weather and climate prediction models and for diagnosis of expected daily ice-cover in Lakes Erie and Superior by operational agencies. Work to produce models for the basins in the other Great Lakes is planned. A description of the models is given in Section 2. Discussion is limited since much of the work was completed just prior to the deadline for this article.

2. TREE-BASED REGRESSION PROBLEM

The goal was to produce a tree-based statistical model for daily prediction of spatially-averaged ice-cover for large basins on the Great Lakes for November to May to be used in numerical weather and climate prediction models, and to provide a means for diagnosis of expected daily to weekly ice-cover by operational agencies. Potential predictors were designed from three considerations: characteristics of winter airmasses and wind conditions affecting a basin, the solar radiation cycle, and the FDD model's daily predictions. The predictand was observed spatially-averaged ice-cover for the winter months December-April of 1965-1979 for each of the six basins defined in Figure 1 for Lakes Superior and Erie. Work to produce models for the other Great Lakes is planned.

For two reasons the task of building a statistical prediction model for ice-cover proved more complex than merely fitting the observed data and using the fit to make daily predictions. The first reason results from the availability of observed ice-cover sporadically rather than every day. Observations for most basins are rather infrequent from late December until mid-late February and more frequent from then to early-mid April, but even then consecutive daily observations are rare. Even though CART fits the observed data very well, there can be large day-to-day fluctuation of the ice-cover predictions for some periods in between the days on which observations are available due to model sensitivity to certain predictors, such as wind speed. At times for some basins large fluctuations actually occur as wind advects the ice, and this is reflected in the observations. Decisions must be made about the extent of day-to-day variability to accept in a prediction model. Another consequence of using a fit of the observed data alone is that large errors in the predictions of ice can occur for the mostly ice-free periods November-December and late April-May for most basins, times for which no observed data is available. Thus the FDD model, which makes a daily prediction for November 1 to May 31, is needed here. The second reason for complication is due to a peculiarity of tree-based statistical regression models, and is called here the "flip-flop problem". It occurs when predictor values are close to the threshold values in the decision trees. The resulting answer can vary wildly from day-to-day because the decision is "flip-flopping" from the left branch to the right branch of the tree. This problem is the most troublesome during periods of rapid changes in ice-cover, but can occasionally occur at any time. The solution to the above all was to produce daily predictions by three methods and use some simple selection rules to produce a final result.

Three sets of tree-based statistical models were produced for each basin: two regression models which fit the observed percent ice-cover data and a six-category classification model which fit the original daily FDD model prediction data. Predictors are explained in Section 3.1. Observed and FDD model data are available as percent ice-cover, ranging from 0-100%. The original FDD model percent ice-cover prediction data was available every day from November 1 1965 to May 31 1983, excluding the summer period. This was converted to six categories: 0%, >0-20%, >20-40%, >40-60%, >60-80%, and >80-100%. One of the tree-based models which fit observed data included the original FDD model ice-cover prediction as a categorical predictor, and one did not. The original FDD model data was fit by CART with atmospheric and solar radiation predictors. The two observed-data CART models and the CART FDD model were run daily from November 1 1965 to May 31 1983. The CART FDD model was used to generate a daily FDD categorical ice-cover prediction as an input predictor to the observed-data model which included the original FDD data as a predictor. This procedure eliminates explicit dependence on the FDD model and thus the need to directly predict surface temperature and to know in advance when the date of maximum freezing degree days would be reached in a winter. The three sets of daily ice-cover predictions were scanned for each basin in order to formulate simple selection rules that yield a single prediction for each basin for each day from November 1 to May 31.

2.1 PREDICTORS

Sixteen predictors were originally tried. Several runs were made trying various combinations of predictors, with the eight predictors shown in Table 1 found to give the best overall results. Linear combinations of predictors were not found to improve the results. Atmospheric predictors were calculated with the U.S. National Meteorological Center (NMC) 47x51 381 km grid-point analysis data for 0000 UTC and 1200 UTC obtained from the National Center for Atmospheric Research (NCAR). A 1000-mb geostrophic wind direction was determined at each analysis time in the center of each basin in order to calculate atmospheric predictors at the nearest onshore basin boundary location upwind from the basin center. Boundary points were distributed approximately every 50-km around each basin. An 850-mb temperature predictor was used in place of surface temperature predictor due to occasional bad surface temperature data in the NCAR data base, particularly in the years 1977-1979, and because surface temperature can be notoriously variable in weather and climate model predictions. The seasonal solar radiation cycle was parameterized with a simple sine function.

2.2 RESULTS

CART initially produces a tree which fits all the data perfectly then finds a series of increasingly less complex trees by systematically reducing the number of decision nodes (known as pruning) until only one node remains. The error of each tree when applied to independent data is estimated for large data sets (more than about 1000 cases) by reserving a portion of the learning data for testing and building the trees with the remaining data, and for small data sets by estimating the error with by cross-validation. CART decision trees were produced for the scenarios described in the introduction to Section 3. Regression trees which fit the observed data ("OBS:FDD-IN" and "OBS:FDD-OUT" models) were constructed with the "least absolute deviation (LAD)" of ice-cover values of cases within a node, while errors in applying these trees to independent data were estimated by "10-fold cross-validation". Classification trees which fit the FDD model data ("CART-FDD") were constructed with the "ordered-twoing" option, while errors when applying these trees to independent data were estimated by reserving 1/3 of the original data sample as a test sample. The final trees selected for use were those found to have the minimum estimated error or close to it in a few cases where that tree had very few nodes.

An error summary for the decision trees is given in Table 2. Errors for the observed-data-fit decision trees are expressed as percent ice-cover, and errors for the FDD model-data-fit decision trees are expressed as the fraction of misclassified data. We see that using the FDD model data as a predictor lowers the fit-error of the trees for all but the Erie-East basin. When using the observed-data-fit trees with independent data, the error of the ice-cover percentage prediction for the Lake Superior basins is estimated to be about 10-20%, and about 15-20% for the Lake Erie basins. This is a respectable result, considering the error in the ice-cover observations themselves is about 10%.

Table 3 shows the importance ranking of predictors. The airmass indicator predictors AVTHK, CUMTHK, and AVTEM850, along with FDDMODEL are the overall most important for all basins. The solar radiation predictor SINEDATE was next in overall importance, and was relatively more important for Lake Erie than Lake Superior. The least important predictors were QAD700, which is related to cloud cover, and the daily wind speed predictor DAYSPEED.

The next step was to make a model for daily ice-cover predictions. The CART-FDD, OBS:FDD-IN, and OBS:FDD-OUT models were re-run for each basin each day from

November 01 1965 May 31 1983. The prediction by CART-FDD provided the categorical ice-cover FDD input predictor for the OBS:FDD-IN prediction, thus eliminating explicit dependence on the FDD model. The daily predictions by all three were scanned for some simple selection rules to provide a final daily prediction model, (CART-SR model). These arbitrary rules are formulated with the basic philosophy of staying close to the FDDMODEL prediction for the November to mid December and late April to end of May periods; using the OBS:FDD-OUT prediction for the early winter freeze-up and spring melt-down periods of rapid ice-cover change, when the FDD model is likely to have large error; using the OBS:FDD-IN prediction for the winter period; and checking that the rules did not increase overall error of the fit of the observed data. The flip-flop problem must also be dealt with in the rules. Table 4 shows the predictions for the Erie-Center and adjacent Erie-East basins for January 11 - February 2, 1976, a period of rapidly increasing ice-cover. Several points are illustrated here. The observations of spatially averaged ice-cover are not continuously available in time. The CART tree-based regression ice-cover prediction values are generally much closer to the observed ice-cover values than are the original FDD model values. The tree-based values jump non-continuously in time as the terminal decision nodes change because the value in each node is a least-absolute-deviation value of the ice-cover of all the cases in the node. This is not always cause for alarm - there is considerable fluctuation in the observations themselves, which the smoothly varying FDD model does not handle, but which is handled by the CART models. The CART-SR predictions for both basins change over from the OBS:FDD-OUT model to the OBS:FDD-IN model in late January. The flip-flop problem struck the Erie-Center CART-FDD and OBS:FDD-IN predictions January 22 and OBS:FDD-OUT predictions January 28-31. The CART-SR model selection rules detected this and switched the CART-SR predictions to the opposite model in both occurrences.

Table 5 shows errors for the fit of the observations by the different models and of the CART fit of the FDD model. Comparing the absolute value errors in column 2 with the numbers in column 3 of Table 2 shows that using the CART-FDD prediction for OBS:FDD-IN model does not substantially affect the accuracy of the prediction. The CART-SR model error is close to that of the OBS:FDD-IN and OBS:FDD-OUT models. All of the CART tree-based models are seen to have substantially less error than the original FDD model.

3. CONCLUDING REMARKS

Two examples of using a tree-based statistical model (CART) to develop diagnostic and prediction models for atmospheric science problems were mentioned. The tree-based classification snowsquall prediction model is already in operational use at the Ontario Weather Center and has been found by the forecasters to be accurate and useful for mesoscale prediction of location and snow amounts from lake-effect snowsqualls. The tree-based regression models for prediction of spatially-averaged ice-cover have 1/3 to 2/3 of the error of Assel's freezing degree day model and are capable of predicting daily ice-cover to within 10-20% for the Lake Superior basins and 15-20% for the Lake Erie basins. These models proved particularly adept at handling ice-cover in the Superior-Whitefish Bay and the three Lake Erie basins, where ice-cover can be highly variable from day to day. Work is planned to develop models for basins on the other Great Lakes.

Tree-based statistical models are a relatively new development. Based on the success of CART for the two quite different applications mentioned here, the use of CART for other applications in the atmospheric and environmental sciences is encouraged.

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TABLES AND FIGURES

Table 1: Potential predictors used to establish regression decision trees for ice-cover on the basins shown in Figure 3.

1. FDDMODEL - Assel (1990) freezing-degree-day model value of percent ice-cover in 6 categories: 0, >0-20, >20-40, >40-60, >60-80, >80-100.

2. AVTHK - average 700-mb to 1000-mb thickness in meters from November 1 to the current day. Slowly decreases from a maximum near November 1 to a minimum in late winter then slowly increases.

3. CUMTHK - cumulated 700-mb to 1000-mb thickness minus 2800 meters from November 1 to the current day. 2800 meters thickness corresponds roughly to a surface temperature of 0 deg C. Same variation as AVTHK.

4. AVZ1000 - average 1000-mb height from November 1 to the current day.

5. AVTEM850- average 850-mb temperature from November 1 to the current day.

6. DAYSPEED - 1000-mb geostrophic wind speed in meters per second over the basin center for the current day.

7. QAD700 - 700-mb advection of absolute vorticity over the center of the basin for the current day. This should be related to middle-level cloud.

8. SINEDATE - the sine of two pi times (the day number from November 1 minus 141). Has negative value before March 21 and positive value after. Varies fastest at spring equinox and slowest at winter solstice. For parameterization of solar radiation.

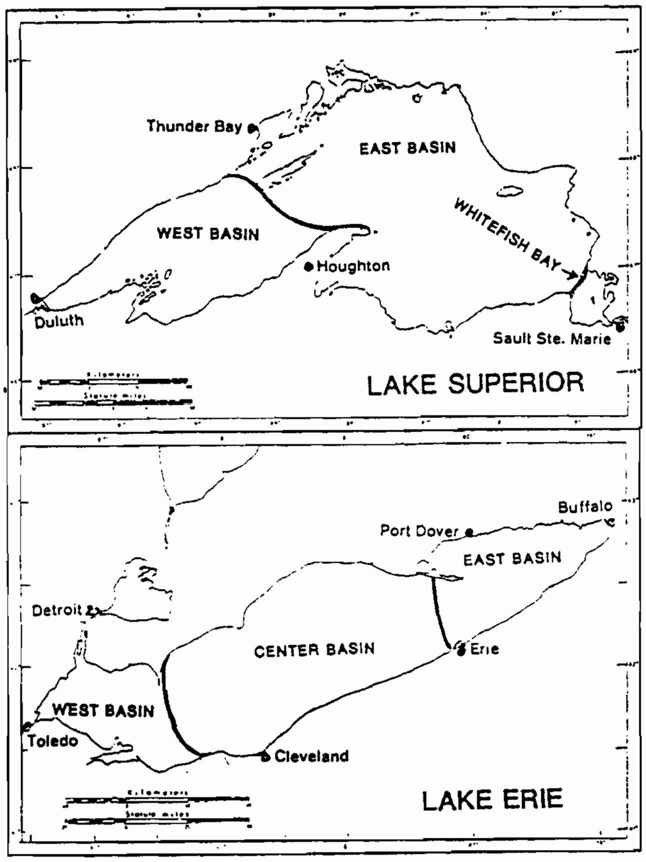

Figure 1: West, east, and Whitefish Bay basins of Lake Superior and west, center, and east basins of Lake Erie, from Assel (1990).

Table 2: For each basin: column (1) number of observations available for growing CART decision trees; column (2) number of terminal decision nodes in trees selected for use; column (3) summary of error of fit of learning data fit; column (4) estimated error when decision trees are used with independent data. Errors for OBS:FDD-IN and OBS:FDD-OUT regression trees are ice-cover percent, errors for CART-FDD classification trees are fraction of events misclassified.

ERIE - WEST				
OBS:FDD-IN	176	38	4.5 %	15.1%
OBS:FDD-OUT	176	28	6.6 %	16.9
CART-FDD	3757	199	.008	.106
ERIE - CENTER				
OBS:FDD-IN	101	30	2.9 %	15.1%
OBS:FDD-OUT	101	28	4.5 %	20.9%
CART-FDD	3757	190	.004	.097
ERIE - EAST				
OBS:FDD-IN	182	22	6.8 %	14.0%
OBS:FDD-OUT	182	28	6.5 %	15.4%
CART-FDD	3757	190	.004	.109
SUPERIOR - WEST				
OBS:FDD-IN	69	14	6.7 %	19.8%
OBS:FDD-OUT	69	17	6.7 %	20.4%
CART-FDD	3757	204	.005	.086
SUPERIOR - EAST				
OBS:FDD-IN	48	21	1.6 %	15.8%
OBS:FDD-OUT	48	20	2.0 %	17.0%
CART-FDD	3757	156	.011	.089
SUPERIOR - WHITEFISH BAY				
OBS:FDD-IN	340	60	2.8 %	9.5%
OBS:FDD-OUT	340	9	7.7 %	10.5%
CART-FDD	3757	209	.004	.084

Table 3: CART ranking of predictors based on the number of times each predictor is used in the total process for finding the best tree. Rankings are on a scale of 0-100. Shown are the predictor rankings for each of the basins.

SUPERIOR WEST

CART - FDD MODEL IN		CART - FDD MODEL NOT IN		CART FIT OF FDDMODEL	
AVTHK	100	AVTHK	100	AVTEM850	100
FDDMODEL	80	AVZ1000	85	CUMTHK	80
AVTEM850	75	AVTEM850	58	SINEDATE	58
AVZ1000	71	CUMTHK	57	AVTHK	38
CUMTHK	54	SINEDATE	53	AVZ1000	29
SINEDATE	49	DAYSPEED	50	QAD700	7
QAD700	40	QAD700	32		
DAYSPEED	18				

SUPERIOR EAST

CART - FDD MODEL IN		CART - FDD MODEL NOT IN		CART FIT OF FDD MODEL	
AVTHK	100	AVTHK	100	AVTEM850	100
FDDMODEL	67	CUMTHK	59	CUMTHK	78
CUMTHK	59	DAYSPEED	57	SINEDATE	65
AVZ1000	58	AVZ1000	49	AVTHK	41
DAYSPEED	51	AVTEM850	46	AVZ1000	24
AVTEM850	43	SINEDATE	39	QAD700	6
SINEDATE	35	QAD700	18		
QAD700	17				

SUPERIOR WHITEFISH BAY

CART - FDD MODEL IN		CART - FDD MODEL NOT IN		CART FIT OF FDD MODEL	
AVTEM850	100	AVTEM850	100	CUMTHK	100
AVTHK	93	CUMTHK	88	AVTEM850	96
CUMTHK	89	AVTHK	88	SINEDATE	70
FDDMODEL	78	SINEDATE	74	AVTHK	53
SINEDATE	77	QAD700	42	AVZ1000	30
QAD700	52	AVZ1000	32		
AVZ1000	38	DAYSPEED	25		
DAYSPEED	29				

ERIE WEST

CART - FDD MODEL IN		CART - FDD MODEL NOT IN		CART FIT OF FDD MODEL	
SINEDATE	100	SINEDATE	100	CUMTHK	100
FDDMODEL	97	CUMTHK	83	AVTEM850	93
CUMTHK	84	AVTEM850	56	AVTHK	90
AVTHK	61	AVZ1000	53	SINEDATE	83
AVZ1000	50	AVTHK	49	AVZ1000	31
QAD700	49	DAYSPEED	34	QAD700	10
AVTEM850	41	QAD700	28		
DAYSPEED	33				

ERIE EAST

CART - FDD MODEL IN		CART - FDD MODEL NOT IN		CART FIT OF FDD MODEL	
FDDMODEL	100	AVTHK	100	CUMTHK	100
AVTHK	84	CUMTHK	88	AVTEM850	74
CUMTHK	80	SINEDATE	83	SINEDATE	67
SINEDATE	56	AVTEM850	75	AVTHK	54
AVTEM850	56	AVZ1000	42	AVZ1000	37
AVZ1000	40	DAYSPEED	21	QAD700	10
DAYSPEED	29	QAD700	21		
QAD700	27				

ERIE CENTER

CART - FDD MODEL IN		CART - FDD MODEL NOT IN		CART FIT OF FDD MODEL	
FDDMODEL	100	SINEDATE	100	CUMTHK	100
CUMTHK	97	AVTHK	93	AVTEM850	73
SINEDATE	94	AVTEM850	90	SINEDATE	60
AVTHK	87	AVZ1000	58	AVTHK	52
AVTEM850	69	QAD700	27	AVZ1000	42
AVZ1000	43			QAD700	14
QAD700	38				
DAYSPEED	35				

ERIE EAST

1	2	3	4	5	6	7	8	9
76 01 11	9999	27	22	22	7	41	4	4
76 01 12	9999	27	22	22	15	43	4	4
76 01 13	9999	27	22	22	22	44	4	4
76 01 14	9	27	22	22	22	45	4	4
76 01 15	25	27	22	22	22	48	4	4
76 01 16	9999	27	22	22	22	49	4	4
76 01 17	74	27	74	74	39	55	4	4
76 01 18	9999	33	22	22	39	61	5	5
76 01 19	9999	33	22	22	39	64	5	5
76 01 20	22	33	22	22	22	64	5	5
76 01 21	9999	33	22	22	22	65	5	5
76 01 22	33	33	30	30	25	70	5	5
76 01 23	9999	33	30	30	27	75	5	5
76 01 24	86	91	86	86	49	78	5	5
76 01 25	9999	91	86	86	67	79	5	5
76 01 26	44	54	52	52	75	78	5	5
76 01 27	52	54	52	52	63	78	5	5
76 01 28	9999	91	86	86	63	80	5	5
76 01 29	9999	94	86	94	77	81	6	6
76 01 30	9999	94	86	94	91	82	6	6
76 01 31	9999	94	86	94	94	84	6	6
76 02 01	9999	94	86	94	94	85	6	6
76 02 02	9999	94	86	94	94	88	6	6
76 02 03	86	94	86	94	94	89	6	6

ERIE CENTER

1	2	3	4	5	6	7	8	9
76 01 11	9999	17	44	17	7	61	5	5
76 01 12	9999	17	44	17	12	61	5	5
76 01 13	9999	17	44	17	17	61	5	5
76 01 14	17	17	44	17	17	62	5	5
76 01 15	9999	17	44	17	17	62	5	5
76 01 16	9999	17	44	17	17	63	5	5
76 01 17	74	79	74	70	35	64	5	5
76 01 18	9999	79	74	74	54	65	5	5
76 01 19	9999	79	74	74	73	66	5	5
76 01 20	72	79	74	74	74	66	5	5
76 01 21	9999	79	74	79	76	66	5	5
76 01 22	79	1	74	74	76	67	5	1
76 01 23	9999	79	74	79	77	69	5	5
76 01 24	9999	79	74	79	77	70	5	5
76 01 25	9999	79	74	79	79	70	5	5
76 01 26	72	64	74	64	74	70	5	5
76 01 27	81	79	74	79	74	70	5	5
76 01 28	9999	97	3	97	80	70	5	5
76 01 29	9999	79	3	79	85	70	5	5
76 01 30	9999	79	3	79	85	71	5	5
76 01 31	9999	89	3	89	82	72	5	5
76 02 01	9999	79	90	79	82	72	5	5
76 02 02	9999	64	90	64	77	73	5	5
76 02 03	90	89	90	89	77	74	5	5

Table 4: For January 11 - February 2, 1976: observations and model values of spatially-averaged ice-cover for Erie-East basin and Erie-Center basin. Numbers in columns as follows: (1) date - year, month, day; (2) observed ice-cover (%), 9999 = no observation; (3) OBS:FDD-IN model ice-cover (%); (4) OBS:FDD-OUT model ice-cover (%); (5) CART-SR model ice-cover; (6) 3-day smoothed CART-SR model ice-cover (%); (7) original FDD model ice-cover (%); (8) original FDD model ice-cover category (1-6); (9) CART fit of FDD ice-cover category (1-6).

Table 5: RMS errors and average absolute value of errors for each basin. Columns: errors for the fits of observed ice-cover data for predictions by: (1) OBS:FDD-OUT model; (2) OBS:FDD-IN model; (3) CART-SR model; (4) running 3-day average of CART-SR model; (5) the origial FDD model; (6) CART-FDD fit of the FDD categorical data.

RMS Errors (Percent Ice-cover)

	1	2	3	4	5	6
SUPERIOR WEST	10.8	10.8	11.9	16.6	24.41	0.26
SUPERIOR EAST	3.7	8.6	7.9	11.4	15.1	0.05
SUPERIOR WHITEFISH BAY	14.8	7.7	8.3	11.5	17.2	0.23
ERIE WEST	11.9	9.7	9.7	11.6	20.0	0.37
ERIE CENTER	7.8	10.8	8.8	12.2	23.3	0.41
ERIE EAST	9.8	12.4	10.0	12.7	18.3	0.37

Average Absolute Value of Errors (% Ice-cover)

	1	2	3	4	5	6
SUPERIOR EAST	2.0	2.8	2.7	6.1	12.5	0.05
SUPERIOR WEST	6.9	6.7	7.6	9.5	21.3	0.03
SUPERIOR WHITEFISH BAY	7.8	3.3	3.6	6.3	10.8	0.03
ERIE EAST	6.4	7.8	6.5	8.5	14.2	0.06
ERIE WEST	6.8	5.5	5.5	7.8	14.9	0.07
ERIE CENTER	4.6	4.9	4.2	7.7	17.7	0.06