

Estimating photosynthetically available radiation into open and ice-covered freshwater lakes from surface characteristics; a high transmittance case study

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Abstract

A simple technique, based on several published studies, is presented to estimate photosynthetically available radiation (PAR: 400–700 nm) at the air/water and ice/water interfaces on freshwater lakes. Grand Traverse Bay of Lake Michigan of the Laurentian Great Lakes before, during, and after ice cover is used as a case study. The technique depends on assigning PAR transmittances to air/water or air/ice surfaces from empirically determined relationships. During ice cover, PAR reaching the water column under the ice exceeded 45% of incoming PAR, on the average, due to the amount of clear ice present on the bay.

Introduction

Growth and survival of freshwater phototrophic phytoplankton in winter depends on the amount of incident photosynthetically available radiation (PAR: 400–700 nm) which penetrates through snow and ice surfaces. In north-temperate lakes of the U.S. and Canada, an ice cover first forms from freezing of water without snowfall to produce clear ice, or from freezing of a slush layer in the presence of snowfall to produce white ice. As the season progresses, snow often accumulates on the ice surface. Additional white ice might form from upward water seepage through stress cracks in the existing ice cover loaded with a thick snow layer, which subsequently refreezes. White ice also forms when mild temperatures or rain reduce a snow cover on the ice to slush, which

subsequently refreezes. Clear ice can continue to form by growth under an initial clear ice layer or under the various forms of white ice. When the ice becomes thick enough to support subsequent snow loads, snow might either accumulate on the ice surface or blow from the ice into the nearshore zone.

On ice-covered freshwater lakes, snow causes the greatest attenuation of incident PAR, with even thin covers (2–8 cm) often reducing transmittances to 10% or less. Depending on crack and bubble structure and cloud cover, clear ice can transmit 70–95% of incoming PAR (Table 1). Transmittances of other ice types are largely bounded by the clear ice/snow-covered-ice transmittance range. An ice-covered lake which is fully blanketed by snow is very different with respect to PAR transmittance and phytoplankton dynam-

Table 1. Typical transmittance (T) measurements from Bolsenga (1978a, 1981, unpublished data).

Ice type	Thickness (cm)			Cloud cover	T (%)
	Clr	Wht	Snw		
Snow Free Ice					
Clear (few crks & bubbles)	36			Clear	95
Clear (many small bubbles)	26			Clear	87
Clear (stippled sfc)	21			Overcast	79
Clear (few crks & bubbles)	28			Ptly Cldy	77-89
Clear (few crks & bubbles)	36			Overcast	77
Clear (few crks & bubbles)	36			Overcast	70-76
Combination (clr, wht)	37	2		Ptly Cldy-Ovcst	66
Combination (clr, wht)	38	4		Ptly Cldy	58
Combination (clr, wht)	18	7		Uniform Thin Clds	47
Combination (clr, wht)	13	7		Overcast	50
White (dark)		27		Overcast	31
White (light)		27		Overcast	15
Snow Covered Ice					
Combination (clr, wht, snw)	37	2	1	Ptly Cldy-Ovcst	18
Combination (clr, wht, snw)	43	3	1	Clear	17
Brash (thick blocks)	56		Trace	Snow-Blowing Snow	8-15
Combination (clr, snw)	28		3	Ptly Cldy	8-12
Combination (clr, wht, snw)	41	4	5-8	Ptly Cldy	6
Combination (clr, snw)	21		7-8	Overcast	1

ics (Wright, 1964; Roulet & Adams, 1984; Stewart & Brockett, 1984) from one which is covered by only clear ice or patches of snow, white ice, and clear ice (Wright, 1964; Saijo & Sakamoto, 1964; Maeda & Ichimura, 1973).

Estimates of PAR transmittance through various types of ice and ice/snow combinations can be useful to the ecologist in primary production studies, in modelling, or for planning winter biological measurements. Such estimates can also be useful in interpreting remote sensing imagery. Some simple techniques to estimate PAR transmittance through freshwater ice where extensive measurements are not possible or practical (due to a remote location or dangerous ice conditions) are described in this paper. A deep-water bay, during one winter period, provides a case study. PAR was estimated at the air/water and ice/water interfaces on the east arm of Grand Traverse Bay, Lake Michigan, USA (162 km² surface area, Fig. 1) before, during, and after ice cover from November 1985 through April 1986.

Methods

Computing incident PAR

Incident total solar radiation (300–3000 nm) was measured at a station approximately 35 km from



Fig. 1. Location of Traverse Bay with respect to the other Great Lakes. See Fig. 2 for coordinates.

the measurement site. Mesoscale variations in atmospheric conditions are common over such distances. Nevertheless, estimates of atmospheric variabilities can be made by taking into account large-scale flow patterns, the location of water bodies, and topographical influences. Hence the accuracy of extrapolating geographically such incident solar radiation measurements can be ascertained (Bolsenga, 1978b). In this study, the physical characteristics and atmospheric influences at the two sites were sufficiently similar that small differences in incoming radiation would not substantially influence the results. Incident PAR was estimated (Table 2) using a regression of PAR on total downwelling irradiance developed by Jerome *et al.* (1983).

Estimating PAR into open water

To compute the amount of PAR entering the water column during open water for a 'moderately agitated' (see below) water surface, the average water surface albedo (ratio of reflected to incident radiation of the total solar spectrum) was estimated using data from Grishchenko (1959) as modified by Cogley (1979). Grishchenko (1959) measured the albedo of water under various cloud conditions, solar altitudes, and wave heights (from 0.1–0.7 m). Using those measurements, Grishchenko also computed monthly albedo for various latitude bands. Those values, as recomputed by Cogley (1979), have been recommended

Table 2. Monthly average incident photosynthetically available radiation (PAR), in Einsteins m^{-2} , computed from incident total solar radiation (300–3000 nm), in $W m^{-2}$ according to an equation developed by Jerome *et al.* (1983).

Month	Monthly total incident PAR (Einsteins m^{-2})
Nov	174
Dec	145
Jan 1986	252
Feb	402
Mar	683
Apr	897

as normals for fair weather assuming scattered light clouds and a moderately agitated sea surface. However, with any large body of water, high winds and accompanying waves and whitecaps are likely to occur frequently.

When the sea surface is rough with many whitecaps, the Grishchenko data do not apply. Then, according to Willis (1971), the albedo for saltwater with whitecaps varies from 12 to 15%. Possible differences between saltwater and freshwater albedos with whitecaps are not considered significant in this study. At windspeeds less than $7 m s^{-1}$, freshwater whitecap coverage is less than 0.1% of the water surface. The fractional coverage shows an abrupt increase as the windspeed increases from approximately 7 to $8 m s^{-1}$ (Monahan, 1969). Therefore, during the open water period, Grishchenko albedos were used to compute the amount of radiation entering the water for the period that the percent frequency of winds was less than $8 m s^{-1}$. When the percent frequency of winds was greater or equal to $8 m s^{-1}$, an albedo of 15% was assumed. In estimating the amount of PAR entering the water column during open water conditions, it is assumed that water reflectivity in the PAR spectrum (400–700 nm) is equal to reflectivity over the total solar spectrum (300–3000 nm). Such an assumption leads to errors in the estimates. Direct measurement of PAR albedos of calm and agitated freshwater surfaces would provide the basis for estimates with improved accuracy, but in their absence the errors from using total solar spectrum albedo are accepted.

Estimating PAR through ice and snow covers

To compute the amount of PAR entering the water column through snow and ice, the ice types (clear, white, brash, etc.), their general abundance and location, and the position of the ice edge were observed from the surface (aircraft or satellite imagery could have been used), and ice thickness and stratigraphy were measured. Estimated PAR transmittances were then assigned to the various ice types and thicknesses, and those were then

weighted according to the spatial coverage of each ice type/thickness.

Since previously collected measurements provide a major tool to estimate these transmittances, it is useful to briefly review some typical transmittances of different ice types (Table 1). Of all the ice types, clear ice provides the greatest transmittance of incident PAR while snow cover causes the greatest diminution. Other ice types and combinations of ice types show transmittances in the range defined by those bounds. It should be noted that the degree of cloudiness can influence the amount of transmitted irradiance particularly through clear ice (Bolsenga, 1981). If large amounts of clear ice are present in an area where the transmittance is to be estimated, some estimate of the degree of cloudiness should be made since PAR transmittances through clear ice on cloudy days can be significantly lower (approx. 20%) than on clear days (Table 1).

A limited number of transmittance measurements of ice types typical of the area were also made during this study using topside and underwater quantum sensors (above and under ice) mounted on a device described by Bolsenga (1981). A two-layer model was also used sparingly, when sufficient data were not available to transfer previous measurements or measurements from this study, to estimate the combined transmittance, $\tau(C)$, of layered ice covers by the equation (Bolsenga, 1981):

$$\tau(C) = \frac{\tau(A)r(B)}{1 - \rho(A)\rho(B)} \quad (1)$$

where:

ρ = reflectance

τ = transmittance

A = upper layer

B = lower layer

Although used on only a limited basis in this study, the model could be used more extensively in other applications.

Computing PAR averaged over the entire water/ice surface

During the months prior to ice formation (November–December) and after breakup (April), the techniques described to estimate PAR to open water were used. During months when ice was present, quarter-month charts of the extent of ice types (clear ice, white ice, snow), and position of the open water, if any, were drawn (Fig. 2) using visual observations. The surface area of the various ice types and water as a percentage of the total study area was then determined from those charts by planimeter. Transmittance of PAR through the various ice types was calculated using observations of ice thickness and stratigraphy combined with: (a) transmittance measurements from previous studies through various types of ice and combination ice/snow surfaces; (b) the PAR transmittance measurements from this study; and (c) the two layer model, with data from (a) and (b). Once transmittances for the various ice types had been established, those estimates were then weighted according to the quarter-month estimates of the extent of those ice types for that period.

Results

Transmittances of open water (%) by month were: November = 83.6, December = 81.4, January = 82.9, February = 86.5, March = 89.3, and April = 90.7. Computed quarter-month transmittances in the 70–80% range, due primarily to open water, prevailed into early February (Table 3). From the January 24–31 period through the March 24–31 period, at least a portion of the area contained ice cover. The first influence of the ice is indicated, in Table 3, by the overall transmittance of the study area which lowered from 82.9 to 79.6% during the January 24–31 quarter-month period. The first ice cover was composed of white ice previously formed elsewhere, transported into the area, and consolidated into a narrow nearshore zone (Fig. 2a).

During the February 1–7 period, a clear ice

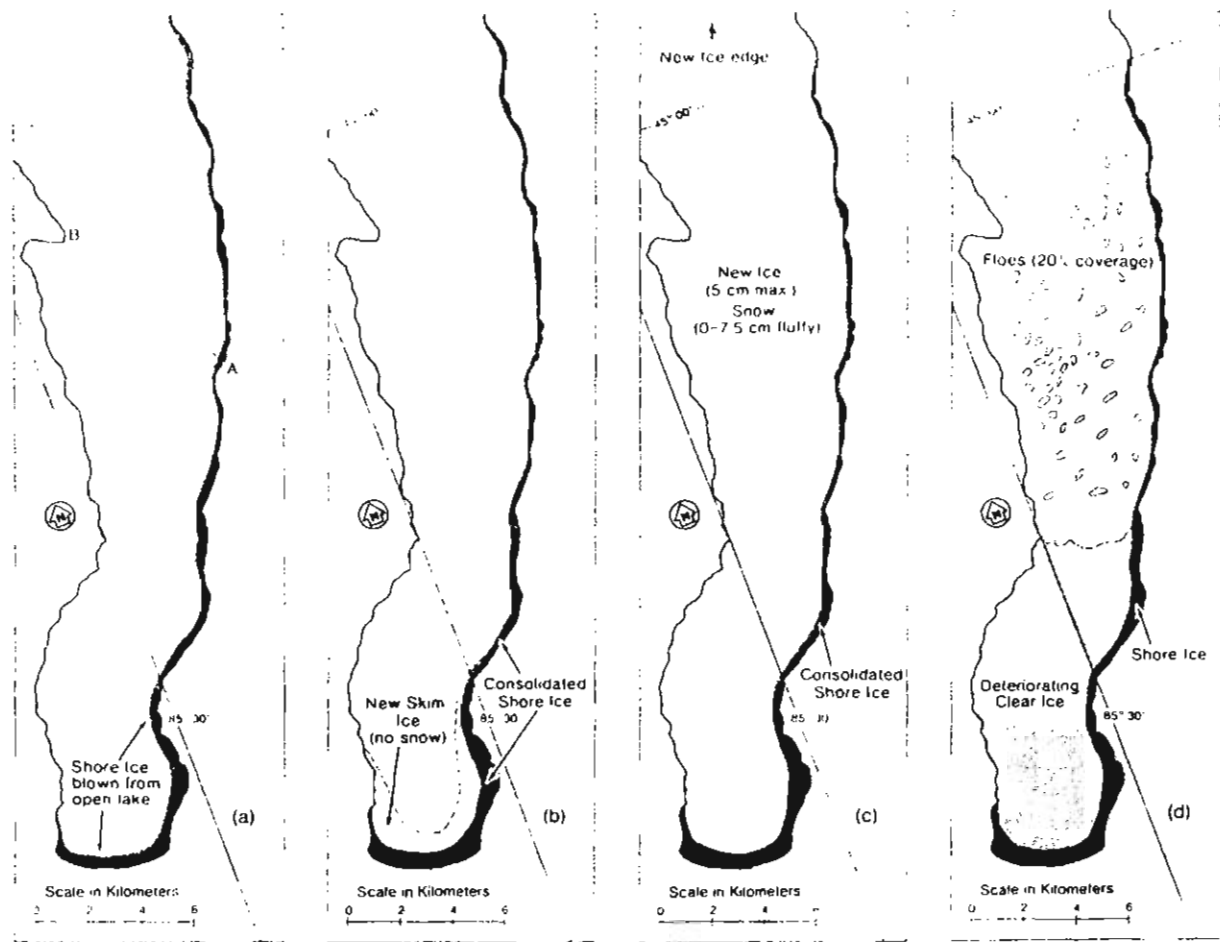


Fig. 2. Average ice conditions during the quarter-month periods (a) January 24–31, (b) February 1–7, (c) February 8–15, and (d) March 24–31.

skim without a snow cover formed (Fig. 2b) causing the overall transmittance to rise slightly from the previous quarter-month period (from 79.6 to 82.8%, Table 3) due to less whitecapping and the high transmittance (80%) of the ice. The average ice extent was estimated at slightly less than 10% (5.2 + 4.4%). On-site observations showed that mostly clear ice covered the entire bay after February 10 (Fig. 2c), but patches of snow began to accumulate on the ice surface which considerably lowered the average transmittance during the February 8–15 quarter-month period (from 82.8 to 43.7%, Table 3). Average ice extent over the period was 100%.

Only one ice thickness measurement, in a near-shore area, was available for the February 16–23

period (8.9 cm wet snow, over 1.3 cm white ice, over 7.6 cm clear ice). This combination was characteristic of the low transmittance ice which comprised portions of the nearshore area. Subsequent thickness measurements on March 2 (Fig. 3) showed that most of the ice in the offshore area was clear ice. Ice surface melting would have been the only process which might have removed white ice layers possibly present during this period. Since significant surface ice melting did not occur between February 16 and March 2, it is assumed for this hindcast of the February 16–23 period that most of the ice on the offshore portion area was of a similar type as that measured on March 2. It therefore consisted of clear ice with some observed overlying snow and some

Table 3. Conditions (ice, open water, or a combination of ice and open water) in the Traverse Bay test area by month or quarter-month periods. The predominant types of ice covering the bay (Type), the percent of the bay covered by each ice type (Cvg), the transmittance (T) of each ice type, and the combined transmittance of the various types of ice for the period weighted according to their coverage (T Ice Tot) are given as well as the coverage (Cvg), transmittance (T), and transmittance of the water surface of the bay weighted by the portion of the bay covered by water (T Wtr Tot). T (%) Bay is the overall transmittance of the bay, obtained by combining T Ice Tot and T Wtr Tot.

Period	Ice			Water			T (%) Bay	
	Type	Cvg (%)	T (%)	T Ice Tot (%)	Cvg (%)	T (%)		T Wtr Tot (%)
Nov.					100.0	83.6	83.6	83.6
Dec.					100.0	81.4	81.4	81.4
Jan.								
1-7	-			-	100.0	82.9	82.9	82.9
8-15	-			-	10.0	82.9	82.9	82.9
16-23	-			-	100.0	82.9	82.9	82.9
24-31	Shore	5.2	20.0	1.0	94.8	82.9	78.6	79.6
Feb.								
1-7	Shore	5.2	20.0					
	Clear	4.4	80.0	4.6	90.4	86.5	78.2	82.8
8-15	Shore	5.2	20.0					
	Clr + Snw	94.8	45.0	43.7	-	-	-	43.7
16-23	Shore	5.2	20.0					
	Clr + Snw	94.8	47.0	45.6	-	-	-	45.6
24-28	Shore	5.2	20.0					
	Mstly Clr + Wht Ice	94.8	52.0	50.3	-	-	-	50.3
Mar.								
1-7	Shore	10.0	20.0					
	Mstly clr + Wht Ice	90.0	48.0	45.2	-	-	-	45.2
8-15	Shore	10.0	20.0					
	Mstly Clr + Wht Ice	90.0	53.0	49.7	-	-	-	49.7
16-23	Shore	10.0	20.0					
	Clear	85.0	80.0	70.0	5.0	89.3	4.5	74.5
24-31	Shore	5.0	20.0					
	Clear	30.8	80.0					
	Floes	20.0	40.0	33.6	44.2	89.3	39.5	73.1
Apr.					100.0	90.7	90.7	90.7

areas of white ice. The estimated transmittance for this period was only slightly higher than that for the preceding period (from 43.7 to 45.6, Table 3).

Weather conditions were highly variable during the February 24-28 period with both wet and dry snowfall, rain, sleet, sunshine, and low and high temperatures causing any snow on the ice to ei-

ther melt or to be transported by winds from ice in the offshore areas to the nearshore areas where the snow then contributed to the formation of white ice. The estimated transmittance increased slightly during the February 24-28 period (from 45.6 to 50.3, Table 3), to account for decreased snow cover, over most of the bay, from the previous quarter-month period.

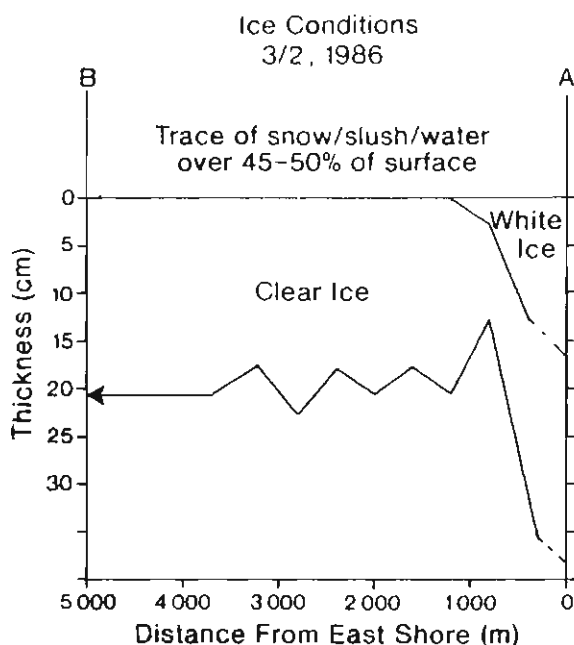


Fig. 3. Ice thickness and stratigraphy on March 2, 1986 along transect A to B shown in Fig. 2(a). Clear ice about 20 cm thick covers the entire offshore area.

On March 2, a traverse was made across the bay (Fig. 2a, from A to approx. 1.75 km from B) drilling boreholes at regular intervals to measure the ice thickness at each location. The measurements showed that the ice in the offshore area was clear, with white ice in the nearshore area (Fig. 3). The clear ice in the offshore areas continued to grow slightly as indicated by measurements on March 4th and 9th. Some snow which accumulated on the offshore ice subsequently contributed to the formation of refrozen slush patches. The area covered by nearshore white ice was estimated to have increased (from 5.2 to 10%) during the March 1–7 period, and with the slush patches offshore, the average transmittance was estimated to be lower than the previous period (from 50.3 to 45.2%, Table 3). Mild temperatures and rain precluded snow accumulation and diminished the slush patch coverage during the March 8–15 period causing an increase in the estimated transmittance (from 45.2 to 49.7%, Table 3). By the March 16–23 period, mild weather conditions had caused ablation of much of the

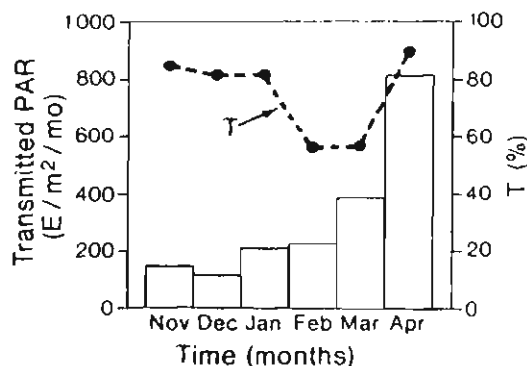


Fig. 4. Transmittances (T) and transmitted photosynthetically available radiation (PAR) during the period of the study.

top of the ice surface. All that remained was clear ice over most of the area as reflected in the increased estimated transmittance (from 49.7 to 74.5%, Table 3). Breakup commenced on March 21 accounting for the small amount of open water (5%) included in the transmittance estimate for this period.

On March 24, the ice cover began to fragment into floes, and on March 26, the floes began to move out of the area. Average transmittance for the March 24–31 period (73.1%, Table 3) reflects 44.2% open water, 5% shore ice, 30.8% clear ice, and 20% floes of candled ice (deteriorating ice with melting along the crystal boundaries) with moderate (40%) transmittance (Bolsenga, 1978a). In the nearshore zone, surface melting of some of the refrozen slush, formed during the March 1–7 period, caused the concentration of low-transmittance shore ice to drop from 10 to 5%. The thicker shore ice, formed during the January 24–31 period, remained in place throughout the period (Fig. 2d).

Transmittances computed on a quarter-month basis were averaged to obtain monthly average transmittances shown in Fig. 4. Those transmittances were then applied to the PAR values in Table 1 to determine the amount of PAR entering the air/water or ice/water interfaces (Fig. 4).

Discussion

The computations indicate that the amount of PAR transmitted to the water column increased

from the months prior to ice cover (November and December 1985 and most of January 1986) to the two months with significant ice cover (February and March 1986). This was due to greater incoming total solar radiation during February and March as compared to November, December, and January in addition to relatively high transmittance values during the ice-covered period.

If November 1985 is taken as a base month for incident total solar radiation ($25\,720\text{ total W m}^{-2}$ for November 1985), December is 18% lower, January is 46% higher, February 135% higher, and March 300% higher than November. Comparing the incident total radiation during the 1985–86 period to 5-year averages shows that values for the 1985–86 period are not unusual, except that the values for November 1985 were lower than normal. Using 5-year averages, December is normally 30% lower, January 19% higher, February 79%, and March 245% higher than November ($31\,318\text{ W m}^{-2}$ for 1981–85 November average). Incident radiation in the area during February and March is apparently usually significantly higher than November and December, not only due to the passage of the winter solstice, but also possibly due to stabilization of weather patterns and decreased cloudiness. Ice cover might contribute to such stabilization by decreasing the heat supply to the atmosphere.

The estimated and measured radiation transmittances during February and March were higher than expected from previous experience and from the literature pertaining to small north temperate region lakes in the U.S. and Canada for two reasons: firstly, larger amounts of clear ice over the bay than expected; and secondly, both ice-covered months include periods of open water. Even though the transmittances during ice cover decreased significantly, transmitted PAR (Fig. 4) does not show the precipitous decreases often observed on snow-covered lakes. High monthly incident total solar radiation, and therefore incident PAR as converted using Jerome *et al.* (1983), during the ice covered months as well as relatively high transmittances contributes to the smooth transition in PAR transmitted to

the water column from fall to winter to spring. Without high incident PAR, transmitted PAR during the ice cover months would have exhibited a significant decrease, but not as much as would be expected for many snow covered inland lakes. The belief that such larger transmittance decreases commonly occur in large lakes has been held by both physical and biological limnologists. It is possible that those assumptions are unfounded for lakes with large wind fetches and that, contrary to published information on small lakes, much of the PAR incident on Great Lakes ice, and possibly on other large lakes in high snowfall areas, is often transmitted through to the water and biota below.

The yearly total 1985–86 snowfall of 111.6 cm was 11.5 cm above the long-term normal. Thus, even with above normal snowfalls, snow on the ice surface of Grand Traverse Bay, and possibly on other large northern temperate lakes, is transported by wind to the nearshore areas. High winds and low temperatures are obviously favorable for this condition. Langlois & Langlois (1985) noted that, on Lake Erie: ‘... dry snow crystals which fall at low temperatures do not fuse readily either with each other or with the sheet-ice, but get blown around by wind, forming drifts’. In this study, the temperatures were usually either very warm causing snow which had accumulated on the ice to melt or very cold, sometimes accompanied by high winds, causing fallen snow to blow off the ice surface to the nearshore zone. Landsat photographs indicate a considerably greater amount of snow on the ice surface in March 1985 (nearly 100% coverage) compared to March 1986. According to weather records at Traverse City, the yearly total snowfall in 1985 was 10 cm higher than in 1986 and 21.5 cm above the long-term normal. Thus, extensive clear ice cover might not be common with snowfalls which are considerably above normal and/or when the proper weather conditions are not conducive to snow blowing or melting.

Marshall's (1966) catalogue of aerial photographs of ice conditions on the Great Lakes and our own field experience indicate that snow falling onto the ice surface often tends to collect in

discrete drifts leaving widespread bare ice surfaces. In smaller inland lakes in high snowfall areas, the ice surfaces tend to be totally blanketed with snow. The point is illustrated by the differences in white ice amounts at two stations separated by 32 km (Fig. 5). L'Anse Bay is connected to the main body of Lake Superior. It is often subjected to high winds which blow snowfall off the ice surfaces of the bay. Portage Lake is an inland lake surrounded by trees and not presumably subjected to as high winds as L'Anse Bay, causing snowfall to accumulate on the ice surface. Yearly total snowfall is 300 cm at L'Anse Bay and 430 cm at Portage Lake. Thus, conditions at Portage Lake seem conducive to producing large amounts of white ice (snow ice as described earlier) with this being less true at L'Anse Bay. White ice comprises nearly 60% of the ice thickness at Portage Lake as compared to 16% at L'Anse Bay (Fig. 5).

Even though there are no data to verify the computed estimates, it is known that several factors can contribute to significant errors in the estimating technique: 1) inaccurate estimates of the overall ice extent; 2) inaccurate estimates of ice type extents and compositions; 3) inaccurate

transmittance estimates; and 4) lack of thickness, stratigraphy, or transmittance measurements. Observations and experienced observers can minimize such potential errors. However, if skillfully applied, it is felt that these techniques can provide acceptable estimates of PAR transmittance through large-area ice and snow covers on lakes where few or no measurements are possible. In addition, it is felt that the hypothesis of higher PAR inputs into larger ice-covered lakes will be strengthened as more measurements in such environments are available.

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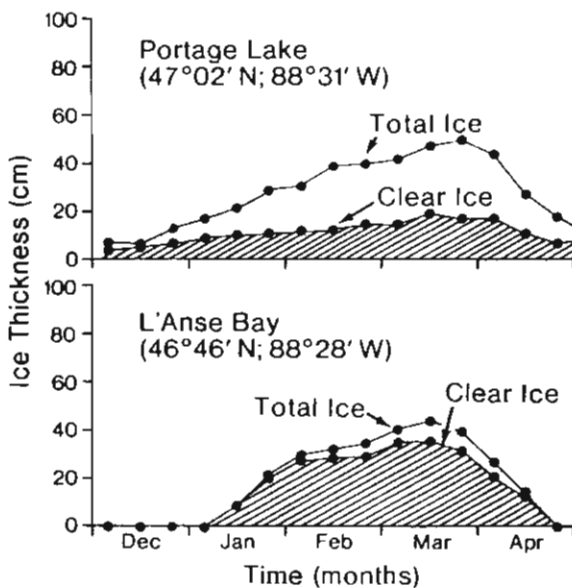


Fig. 5. Variations in white ice at two nearby stations (from Bolsenga, 1988).

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