Proceedings of the Sixth IAHR International Symposium on Stochastic Hydraulics, Taipei, 1992

CHAOTIC DYNAMICS AND OCEAN WAVE STATISTICS[1]

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ABSTRACT : Recent developments in chaotic dynamics have indicated that qualitative information of a dynamical system can be extracted from the observation of a single time series as the time series bears the marks of all other variables relevant in the underlying dynamics. In this paper we review and explore the application of this approach in connection with the study of ocean wave statistics.

INTRODUCTION

The study of waves at the ocean surface has always relied on the concepts that the sea surface waves are random processes, and the surface displacement at a given point can be regarded as the resultant of many independent wave components. Fourier transform and energy spectrum analysis have been frequently used as the efficient and basic method for analyzing measured surface wave time-series data. An effective application of the spectrum analysis requires the assumption that the processes be stationary which is, however, not always realistic. Furthermore, statistical analysis of time series wave data provides little information toward the understanding of the dynamical processes. Recent developments in chaotic dynamics have advanced interesting new approaches for the analysis of time series data. In particular, theorems have been developed that lead to procedures for reconstructing a dynamical system from the observation of a single variable. It appears that a time series actually bears the marks of all other variables relevant in the underlying dynamics, and key features of the dynamics can be extracted from a given time series. In this paper we review and explore the chaotic dynamics approaches and their applications to the study of ocean wave statistics and dynamics. Specifically we examine if it is possible to identify an attractor for an ocean wave time series and determine its dimensionality as well as the minimal dimensionality of the phase space within which the attractor is embedded.

DYNAMIC SYSTEMS AND PHASE SPACE

The advances of chaotic dynamics provide new and stimulating approaches that can be applied to the study of ocean waves. The basic proposition is that relatively simple systems of coupled nonlinear first-order equations often have chaotic solutions. These solutions -- sometimes called strange attractors -- are much more irregular than solutions of traditional dynamic equations. This has generated the hypothesis that some of the fluid flow problems can be qualitatively explained by models that are highly simplified in comparison with full hydrodynamic equations. While the

[1]GLERL contribution No. 786

dynamic systems approach has not yet achieved much quantitative predictive power at the present, it has provided a significant new direction of study.

Following the historical work of Lorenz (1963) a dynamic system can be regarded formally as:

$$\frac{dy}{dt} = F(y), \tag{1}$$

where time, t, is the single independent variable, $y = (y_1, y_2, \ldots y_n)$ represents a state of the system and may be thought of as a point in a suitably defined space -- usually called phase space, and the vector field F(y) is in general a nonlinear operator acting on points in the phase space. A state which is varying in accordance with (1) is represented by a moving particle traveling along a trajectory in phase space. The trajectory becomes a strange attractor when it is chaotic, sensitively depends on the initial conditions, and is attracted to a bounded region in phase space.

Many current studies on chaotic dynamics have focused on the understanding of scaling properties and characterization of strange attractors. Strange attractors can be generally characterized through quantities like Kolmogorov entropy, Lyapunov exponents, and generalized dimensions. If the governing equations are known, then there are reliable methods for determining these quantities. If, however, only measurements of time series are available, then the problem becomes much more cumbersome. In the present study we pursue this latter course of time series analysis without resorting to the knowledge of ocean wave hydrodynamics. We are primarily interested in the applicability of the various approaches of characterizing strange attractors and in particular the determination of dimension of the attractor as the basic degrees of freedom of the system that govern the quantitative predictability of the dynamic system.

DIMENSIONS OF A TIME SERIES

Dimension is one of the most basic properties of geometric objects. Basically, the dimension of a space is the amount of information needed to specify points in the space accurately. For dynamics the dimension provides an indication of the number of essential variables required to represent the dynamics. The dimensionality of a phase space, since it controls the number of possible states, will therefore be associated with the number of a priori degrees of freedom of the system. For chaotic attractors the dimension usually takes on noninteger values. Following the presentations of Atmanspacher et al. (1988), the concept of a fractal (noninteger) dimension D (Mandelbrot, 1982) of an attractor in a d-dimensional phase space with D < d can be deduced from 'information theoretical' considerations:

$$D = \lim_{r \to 0} \frac{I_r}{\log(1/r)}, \tag{2}$$

where the dimension D describes how the information I_r scales with varying spatial resolution r. By dividing the d-dimensional phase space into M boxes of size r^d then the probability that one of the N total points on the trajectory falls into the ith box is $p_i = N_i/N$ and a generalized information of order q is

$$I_q = \frac{1}{1-q} \log\left[\sum_{i=1}^{M} p_i^q \right]. \tag{3}$$

Substituting (3) into (2) leads to the form of generalized dimension defined as:

$$D_q = \frac{1}{q-1} \lim_{r \to 0} \frac{\log\left[\sum_{i=1}^{M} p_i^q \right]}{\log(r)}. \tag{4}$$

Here D_q is a non-increasing function of q, i.e., $D_q \leq D_{q'}$ for all $q \geq q'$. For q equals 0, 1, and 2 the corresponding D_0, D_1, and D_2 are the frequently used fractal, information, and correlation dimensions, respectively.

RECONSTRUCTION OF AN ATTRACTOR

Perhaps one of the most interesting and enticing results developed from the chaotic dynamics is the notion that it is possible to reconstruct certain properties of an attractor in phase space from the time series of a single variable. Following the earlier works of Packard et al. (1980) and Takens (1981), the basic principle is to create a set of m-dimensional vectors from a single time series $x_i = x(t_i)$, $i = 1,\ldots,N$, with the x_i corresponding to measurements in time. This process is known as 'embedding', and m is the 'embedding dimension'. The reconstruction is accomplished by introducing a time lag, s, such that the m-dimensional vectors have the form

$$\mathbf{X}_i = [x(t_i), x(t_i + s), \ldots, x(t_i + (m-1)s)]. \tag{5}$$

In principle, the various characterizations -- the Kolmogorov entropy, the Lyapunov exponents, and the generalized dimensions -- are all accessible through this reconstruction (Simm et al., 1987).

To evaluate the generalized dimension from the attractor reconstruction of a single variable time series, Pawelzik and Schuster (1987) obtained the following definition for D_q corresponding to (4):

$$D_q = \lim_{r \to 0} \frac{\log[C_q(r)]}{\log(r)}, \tag{6}$$

where $C_q(r)$ is the generalized correlation integral of order q given by

$$C_q(r) = \left\{ \frac{1}{N} \sum_{i=1}^{N} \left[\frac{1}{N} \sum_{j=1}^{N} \Theta(r - |\mathbf{X}_i - \mathbf{X}_j|) \right]^{q-1} \right\}^{1/(q-1)}, \tag{7}$$

where $\Theta(r - |\mathbf{X}_i - \mathbf{X}_j|)$ is the Heaviside function which serves to count pairs of points $(\mathbf{X}_i, \mathbf{X}_j)$ that fall within the scale r. For q = 2, (7) reduces to

$$C(r) = \frac{1}{N^2} \sum_{i=1}^{N} \sum_{j=1}^{N} \Theta(r - |\mathbf{X}_i - \mathbf{X}_j|), \tag{8}$$

which is the original correlation integral introduced by Grassberger and Procaccia (1983) for the widely used approach for estimating correlation dimension from a time series data. A detailed review and analysis of this method is given by Theiler (1988).

APPLICATION TO OCEAN WAVE TIME SERIES

Along with the embedding reconstruction measure described in (5), equations (8) and (7) form the basis for the dimensional analysis of experimental time series. While it is relatively computation intensive, the implementations of (8) and (7) are basically straightforward. To test their pertinence in connection with the study of ocean waves, we apply these approaches to the analysis of measurements of wind waves in the Great Lakes. To facilitate the exploratory study, we selected two sets of wind wave data recorded in Lake Erie during 1981 (Schwab et al., 1984). The two data sets comprise generally similar statistical properties. One set, the #3188, recorded a significant wave height of 1.2 m at hour 9:30 on 3 October, whereas the other set, the #4213, recorded a lower significant wave height of 0.3 m at hour 18:00 on 24 October. The calculated energy spectra for the two wave data sets are shown in Figure 1.

We first apply equation (8) to calculate the correlation integral C(r) for embedding dimension m ranges from 4 to 23 as shown in Figures 2(a) and 3(a) for data sets #3188 and #4213 respectively. The left most curve is for $m = 4$ with the subsequent curves for increasing m plotted toward right. The scale resolution r ranged from 0.001 to 10, and we used a time lag of $s = 1$. The slopes of the main part of the C(r) vs. r curves represent the correlation dimension D. The variations of D as a function of embedding dimension m are plotted in Figures 2(b) and 3(b). It is clear that D converges for #3188 beyond $m = 16$. For #4213, however, D converges between $m = 6$ and $m = 12$ but tends to diverge beyond that. We are not certain at the present what caused these differences. We find it is appropriate to choose m equals 18 and 8 respectively for #3188 and #4213 for the calculation of generalized correlation integral $C_q(r)$ using equation (7). Figures 2(c) and 3(c) present the results of $C_q(r)$ vs. r for information order q varies from -9 to 9. The lowest curve in the figures is for $q = -9$. Again the slope of the main part of the curves represents the generalized dimension D_q. Figures 2(d) and 3(d) present the spectra of D_q as a function of q. These smooth spectrum curves for D_q clearly show that generalized dimension exists for wind wave data, and that they are not random processes. It is also of interest to note that while the two data sets may have similar statistical properties, they have significantly different chaotic contents. It appears that waves with higher wave heights have higher D_q spectrum than those with lower wave heights. Furthermore, it is also evident that it is unrealistic to expect a single, unified dimension for all ocean wind waves.

CONCLUDING REMARKS

With this brief review and exploratory study of the application of chaotic dynamics and its approach to ocean wind waves, we find the approach is of interest, useful, and provides significantly new insights toward further understanding of the ocean wave processes. The different generalized dimension for different stages of wave growth resulted from this study, if it can be confirmed from additional studies, will certainly tend to transform the concept of the existence of an universal wave spectrum. The application of the chaotic dynamics approach to the study of ocean wave statistics clearly warrants further detailed and concerted investigations.

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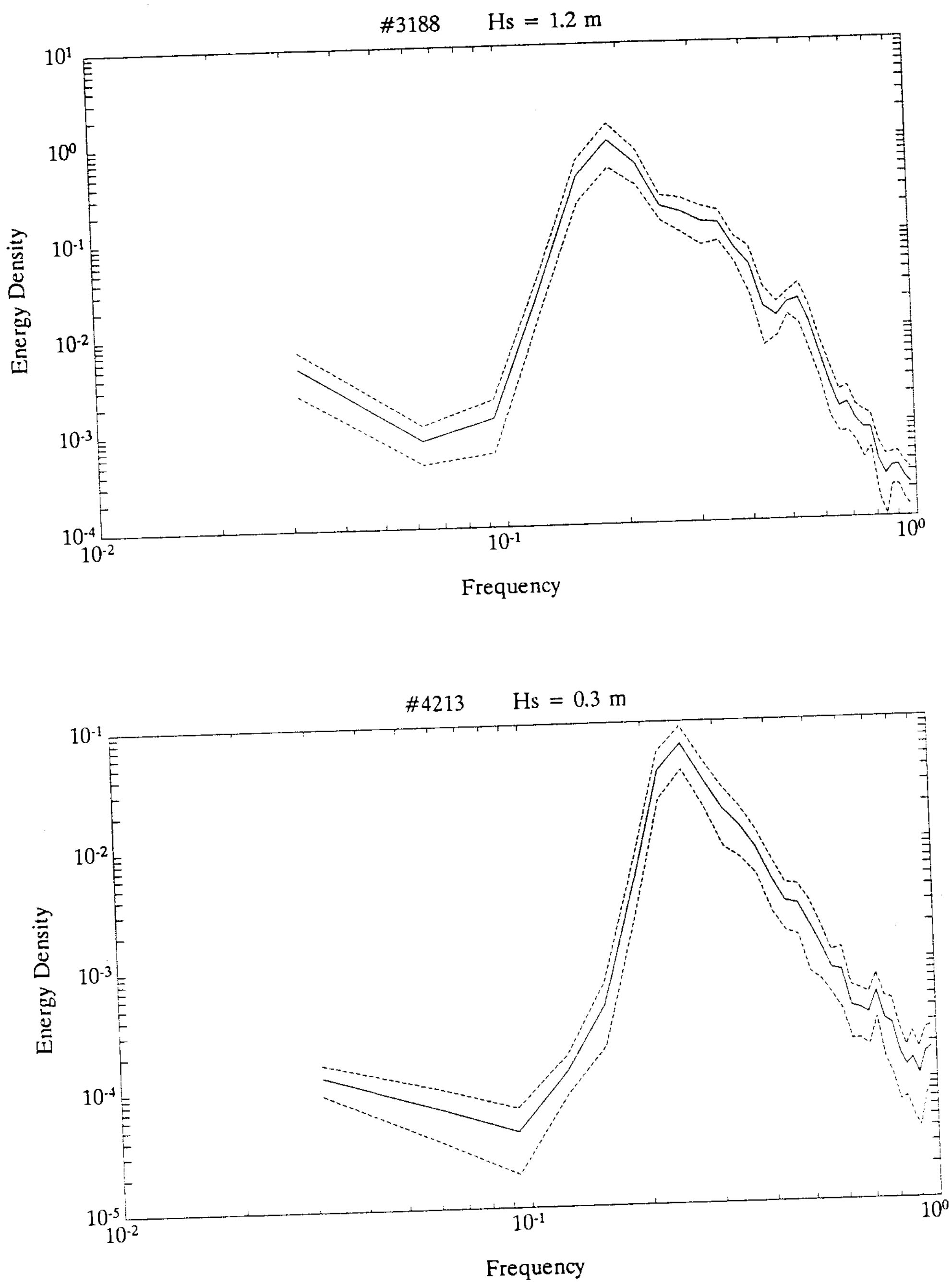

Figure 1. Calculated wind wave energy spectra for the two selected data sets recorded in Lake Erie during 1981.

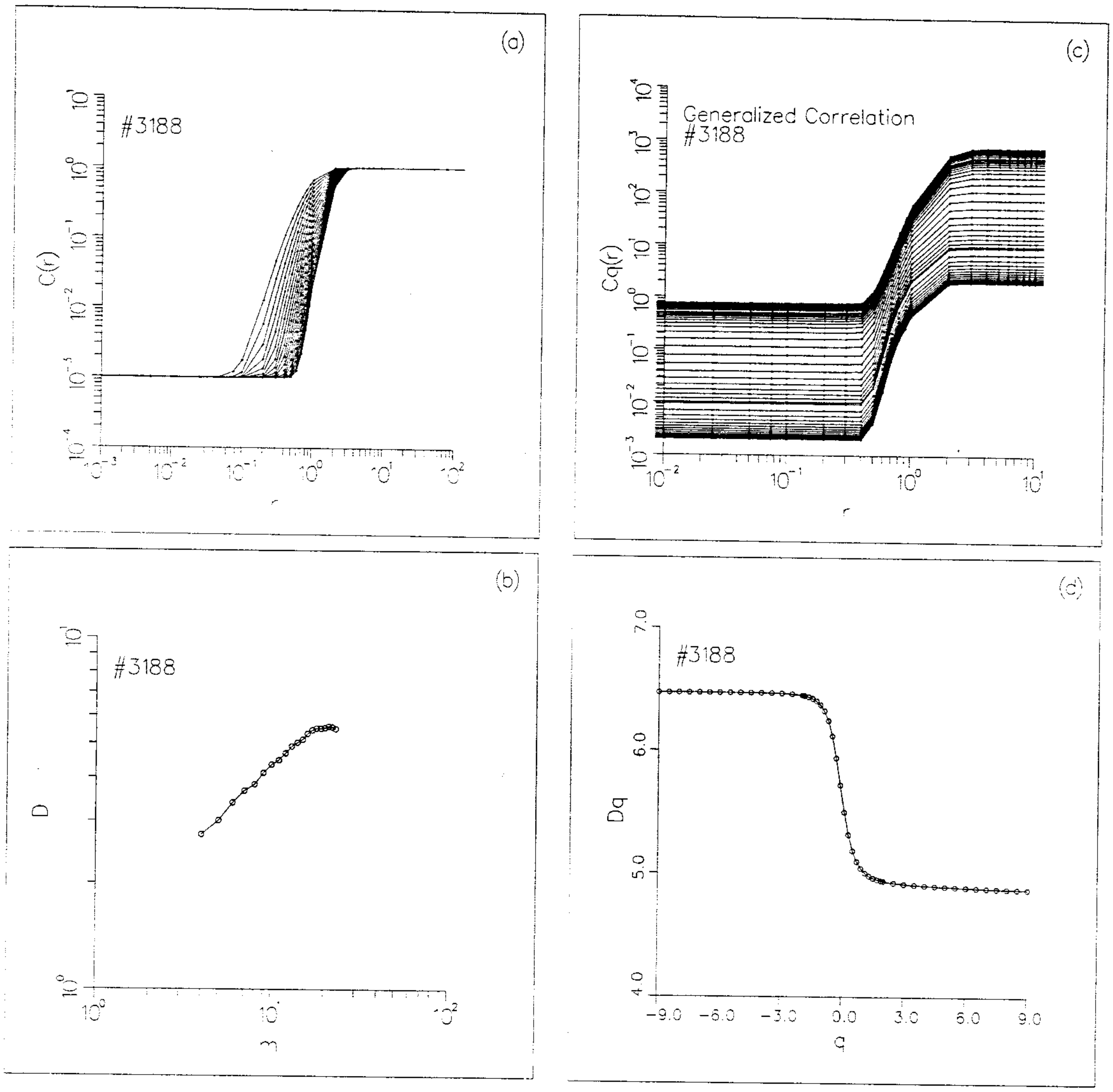

Figure 2. For data set #3188:
(a) Correlation integral C(r) versus scale resolution r for different embedding dimensions m.
(b) The slope of the curves in (a) versus m.
(c) Generalized correlation integral $C_q(r)$ versus r for different information order q.
(d) Generalized dimension D_q versus q.

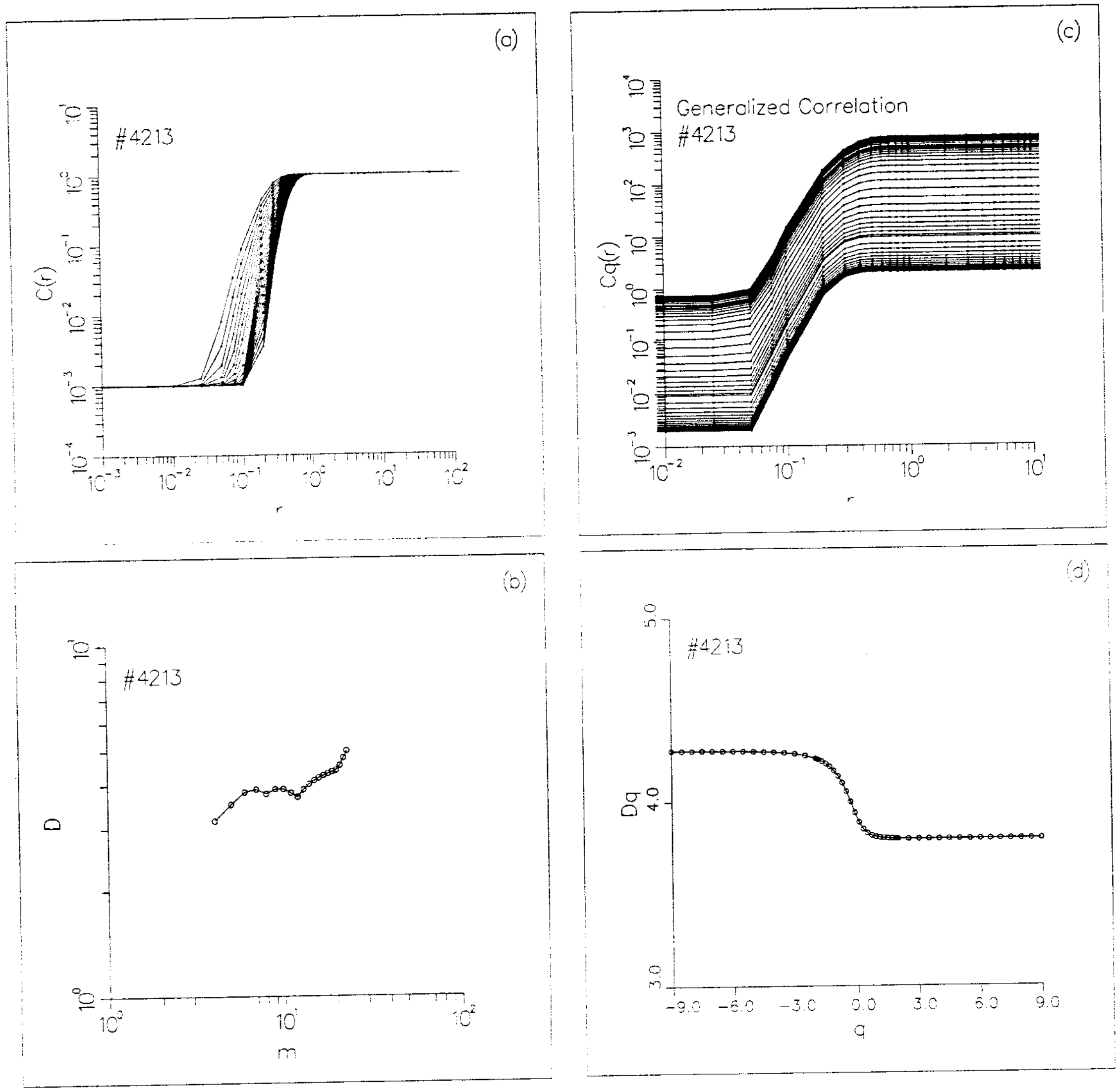

Figure 3. Same as Figure 2 for data set #4213.