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Letter to the Editor

Estimating breaking wave statistics from wind-wave time series data

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Abstract. Wave breaking is a familiar phenomenon which occurs intermittently and ubiquitously on the ocean surface. It is instantly visible from the usual appearance of the whitecaps, yet it is not amenable for routine measurement with customary instruments. Wave breaking has been recognized as playing a crucial role in accurate estimations of the exchange of gases between the ocean and the atmosphere (Wallace and Wirick, 1992) and in the transfer of momentum from wind to the ocean surface (Agrawal *et al.*, 1992). Most of the practical works on wave breaking (Banner and Peregrine, 1993), both in the laboratory and in the field, have been done with specialized methods based on radar reflectivity, optical contrast, or acoustic output of the ocean surface. In this letter I report an application of the wavelet transform analysis (Combes *et al.*, 1989 and Daubechies, 1992) to conventional wave gauge measured time-series data which facilitates the use of classical criterion for distinguishing breaking from non-breaking waves. This simple and fairly efficient approach can be readily applied for an indirect estimation of wave breaking statistics from all available time-series data of wind-generated waves.

Oceanic scientists studying wind waves usually employ two basic approaches to the analysis of time series wave data: statistical analysis using the zero-crossing method (e.g. Anonymous, 1984) and spectrum analysis using Fourier transforms (Kinsman, 1965). While these methods are immensely useful and have dominated the basic wave data analyses for the past four decades, it is also well-known that they are limited to the general characterization of waves over the length of the data set. Besides the recorded surface elevations, they are incapable of providing localized, time-varying information. Most measured wind wave time series data undoubtedly contained breaking waves, but they were ignored in the early analyses and remain undisclosed because of the limitations in the analysis techniques.

Wavelet transform, a recently advanced mathematical technique, has been used successfully for numerous applications including the analysis of signals and images (Combes *et al.*, 1989). The basic assertions of this integral transform is to analyze the time varying signals with respect to both time and scale. The scale decomposition is achieved by translation and dilation of an elementary waveform, referred to as the analyzing wavelets. The wavelet transform of a data set, resulting in a two-dimensional function of time and scale, is simply the inner product between the data and the elementary analyzing wavelet. In practical applications, in general, each scale can be rendered to associate with a corresponding frequency, and the wavelet transform leads to an energy distribution in both frequency and time domain. Thus, it essentially provides a time-varying, localized energy spectrum for each data point in a given time-series. The availability of localized frequency distribution associated with each data point signifies the advent of a new perspective for wave data analysis.

One of the most frequently used approaches for the study of wave breaking is the use of a limiting value of the wave steepness beyond which the surface cannot sustain (Longuet-Higgins, 1969). Alternatively, assuming a linear dispersion relationship, the wave surface will break when its downward acceleration exceeds a limiting fraction γ of the gravitational acceleration, g, that is $a\sigma^2 \cong \gamma g$. This classic criterion can now be easily evaluated for a time series of wave data, as the local wave amplitude a is available from the measured time series, and the local wave frequency σ becomes available from the application of wavelet transform. In classical studies, it has generally been assumed that $\gamma = 0.5$. Recent laboratory studies (Hwang *et al.*, 1989) have shown that γ is closer to 0.4. Some field measurements (Holthuijsen and Herbers, 1986) further indicate that the value of γ should even be lower. In this study I chose to follow the laboratory results and use γ = 0.4.

While the wavelet transform provides a local frequency spectrum for each amplitude data of the wind wave time series, it is not immediately clear which frequency should be used for σ in calculating $a\sigma^2$. Because breaking events are generally associated with the high frequency part of the spectrum, I chose to define σ as an average frequency (Rice, 1954) over the high frequency range, $\lambda f_p - f_n$, of the localized spectrum as

$$\sigma = \left[\frac{\int_{\lambda f_p}^{f_n} f^2 \Phi(f) df}{\int_{\lambda f_p}^{f_n} \Phi(f) df} \right]^{1/2}$$

where $\Phi(f)$ is the localized frequency spectrum given by the wavelet transform, f_p is the localized peak frequency, f_n is the cut-off frequency, and λ is a number greater than 1 that denotes the start of the high frequency range beyond the peak frequency. The exact location of this high frequency range has not been clearly defined. Considering this range as corresponding to the familiar equilibrium range, the frequently used value of λ has been 1.35 or 1.5.

To test this approach, I used a set of wave data measured during the recent SWADE (Surface Wave Dynamics Experiment) program (Weller *et al.*, 1991). The wind and wave data were recorded from a 3-m discus buoy during the severe storm of October 26, 1990. The buoy was located at 73°-48.9'W and 38°-11.6'N, at the edge of the continental shelf offshore of Virginia in the Atlantic Ocean. Wave time series were recorded at 1 Hz from a combined design of a three-axis accelerometer and magnetometer along with the Datawell Hippy system. A total of 100 data sets, each of 1024 s in length, were used in this analysis. The data, predominantly wind sea, covered the entire duration of the storm with wind speeds ranging from calm to 18 m/s and significant wave heights approaching 7 m.

The wavelet transform of the data was computed by means of the complex-valued Morlet wavelet (Farge, 1992), which is a plane wave modulated by a Gaussian envelope that has been successfully applied to the study of sound patterns. While theoretically the Morlet wavelet is only marginally admissible as a wavelet basis, its convenient formulation and its localized frequency independence from time make it advantageous for wind wave studies.

Figure 1 presents an illustration of the analysis where estimated breaking waves are marked on the 256 s long time series segment. The closed circles and open circles represent the results with a high frequency range between 1.35 and 1.5 times that of the local peak energy frequency, f_p respectively, and cut-off frequency, f_n. Clearly with the same cut-off frequency, the lower end of the frequency farther away from the local peak frequency, the open circles lead to higher local average frequency and yield more breaking waves than the closed circles. While the value of 1.35 or 1.5 has been chosen rather arbitrarily from empirical studies, the exploration of breaking waves could potentially serve to resolve the definition of the well-known but still not yet well-defined equilibrium range.

An integration of the wavelet transform over time leads to the familiar energy spectrum. An integration of the wavelet transform over frequency, on the other hand, provides a temporal variation of the localized total energy as shown in Figure 2. The localized energy variations clearly distinguish the groupings of wave data. It is of interest to note that the local peaks of these energy variations also closely correlate with the breaking waves.

Figures 3 and 4 present plots of overall percentages of breaking waves from all the data analyzed in this study as a function of wind speed and significant wave height respectively. While the data points are scattered considerably, it is shown in the results of Figure 3 that there is an approximate linear trend indicating an increase in the percentage of breaking waves with an increase in wind speed. However a similar trend does not hold for significant wave height, as shown in Figure 4, even though significant wave height also generally increased with greater wind speeds. The results

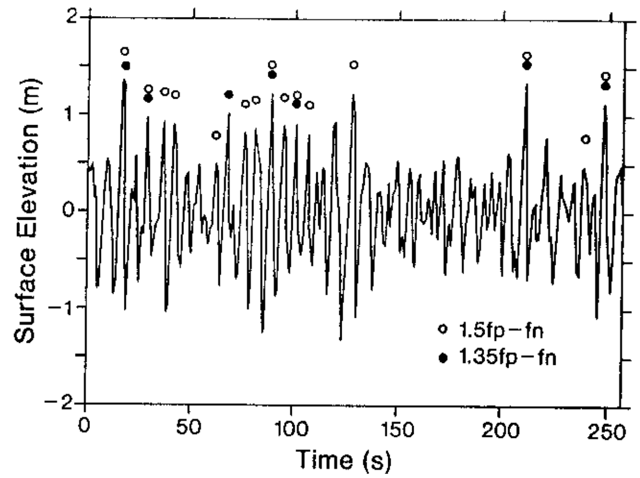

Fig. 1. A segment of sample wind wave time series with possible breaking waves marked by closed circles and open circles that represent two different high frequency ranges as shown. The significant wave height for this data set was 2.5 m under a wind speed of 10.5 m/s.

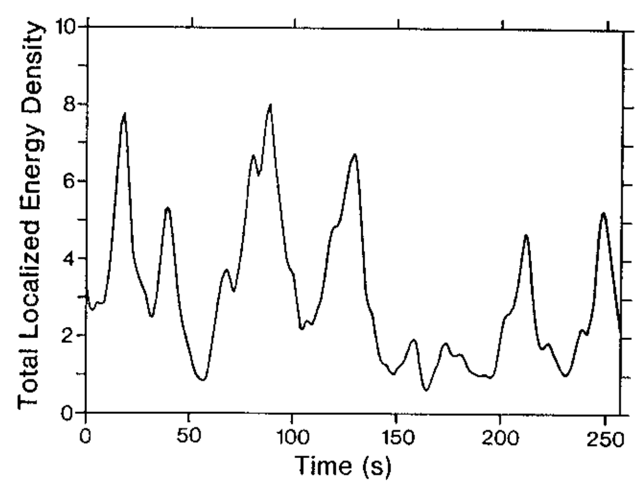

Fig. 2. Temporal data of total localized energy density corresponding to the data segment of Fig. 1. This plot is a result of integrating the wavelet transform of the wind wave time series with respect to frequency. Note that the local peaks of the energy densities correlate with the breaking waves shown in Fig. 1.

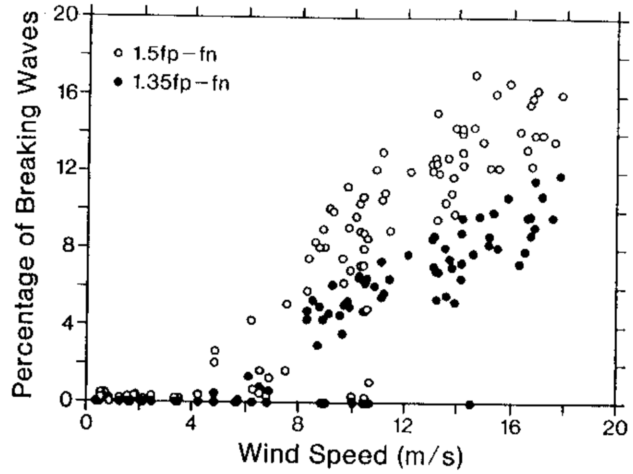

Fig. 3. Plot of the percentage of breaking waves with respect to wind speeds. There is an approximate trend showing the percentage increases with increasing wind speed. The breaking waves start to appear when wind speed exceeds 5 m/s.

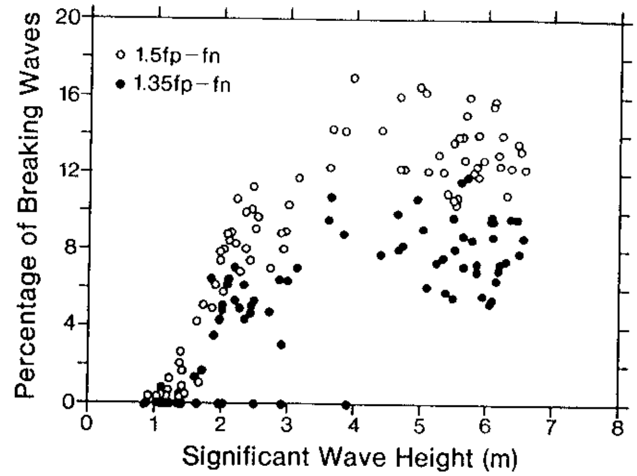

Fig. 4. Plotting of the percentage of breaking waves with respect to significant wave heights. The breaking waves start to appear for significant wave heights greater than about 1m. There is no correlation between breaking waves and higher values of significant wave height for wave height greater than 3 m.

shown in Figure 3 are in general accord with various available observations (Holthuijsen and Herbers, 1986). According to these results, breaking waves become prevalent when wind speeds exceed 4 m/s and significant wave heights rise beyond 1 m.

At the present, the limiting fraction of downward wave acceleration from the gravitational acceleration, γ, and the parameter locating the local equilibrium range beyond local peak frequency, λ, are both tentative. Therefore, the wavelet transform approach that leads to these results is useful, convenient, and also exploratory. Perhaps a superior simultaneous measurement of wind-wave time series and wave breaking would suffice to substantiate the approach. Unfortunately operative and sufficient instrumentation for this simple purpose is still wanting.

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