NOTES AND CORRESPONDENCE

On the Net Cyclonic Circulation in Large Stratified Lakes

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ABSTRACT

This paper proposes a possible explanation for the mean cyclonic circulation in large stratified lakes. The condition of no heat flux through the bottom boundary causes the isotherms to dip near the shores to intersect the sloping bottom orthogonally. This "doming" of the thermocline causes an internal pressure gradient in the surface layer with higher pressure nearshore and results in a geostrophic cyclonic circulation.

1. Introduction

As reported by Emery and Csanady (1973), many large stratified lakes and semienclosed marginal seas and estuaries exhibit a persistent cyclonic circulation. This pattern is most apparent during the period when the lakes are stratified. This background circulation is generally small compared to the transient circulation patterns set up by winds. But because of the persistence of this background circulation, and because the wind-induced currents are often transitory and tend to cancel out in the mean, this background circulation can be responsible for the long-term movement and redistribution of dissolved and suspended material in the water body, as well as the temperature structure.

Several explanations for this mean cyclonic circulation in stratified basins have been proposed. Emery and Csanady (1973) suggested a mean cyclonic curl in the wind stress field as a possible mechanism. The cyclonic curl results from the asymmetry of the surface water temperature field in a stratified lake when it is exposed to a uniform wind field. Wunsch (1973) proposed that the Lagrangian drift associated with large internal Kelvin waves in a stratified lake might account for the net cyclonic drift. Bennett (1975) also invoked an asymmetric response of a stratified lake to a uniform wind to explain the net cyclonic circulation, but in his proposed mechanism, enhanced vertical stratification on the downwelling shore decreases vertical mixing and bottom friction, resulting in stronger currents on that shore. Bennett (1977) compared all of these mechanisms to results obtained using a three-dimensional numerical model with variable horizontal grid size. His conclusion was that higher grid resolution near the shores was more important than changes in the atmospheric forcing or the turbulence formulation in improving the model's ability to simulate the long-term mean flow. Simons (1986) discusses how the rectified effects of nonlinear topographic wave interactions in a barotropic lake can also be responsible for a mean cyclonic circulation.

The purpose of this note is to examine another mechanism that may contribute significantly to the mean cyclonic summer circulation in large lakes. This mechanism arises from the boundary condition of no heat flux through the bottom of the lake. This boundary condition requires that there is no temperature gradient normal to the bottom, so that the isotherms will intersect the bottom orthogonally. The effect of this boundary condition on a stably stratified lake with a sloping bottom is to impart a dome-shaped configuration to the thermocline—that is, the

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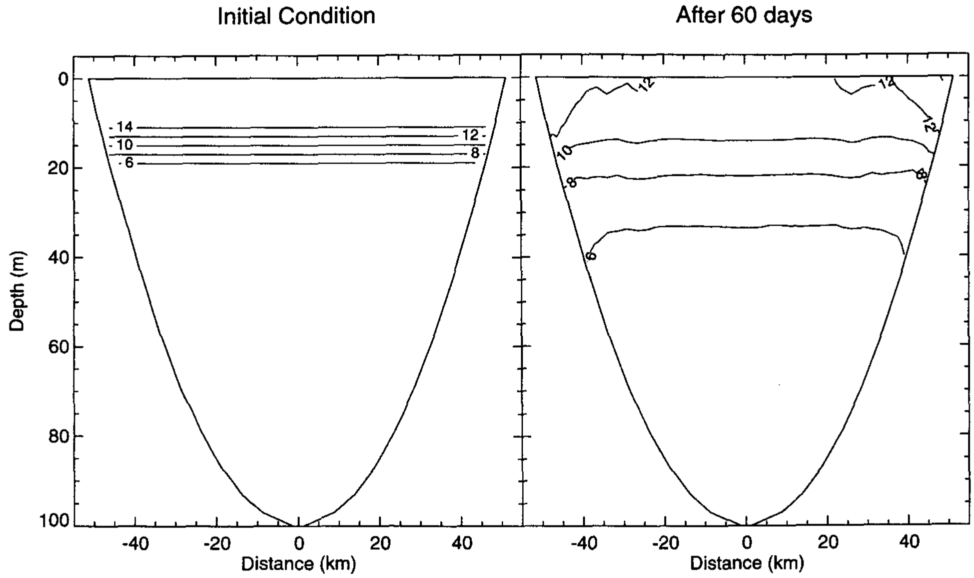

FIG. 1. Initial and final (after 60 days) temperature distribution in a cross section of a circular basin with no surface heat flux and no heat flux through the bottom.

thermocline is deeper near the shore and shallower in the deeper regions of the lake. A cyclonic circulation is set up to maintain geostrophic balance with the pressure gradient field. This effect is illustrated using a full-featured three-dimensional numerical circulation model.

2. The model

The Navier–Stokes equations of fluid mechanics govern the water motions in a stratified lake. We assume the flow is hydrostatic and that the Boussinesq approximation applies. We consider a stably stratified fluid with no wind stress, no surface heating, and no heat flux through the bottom of the basin. In the case of a flat bottom, thermal diffusion mixes the stratified fluid to an equilibrium state of uniform temperature and no motion. At all times, isotherms are horizontal and there is no horizontal motion. When the basin has a sloping bottom, however, the bottom boundary condition of zero heat flux precludes a steady-state motionless solution. Even with simple geometry, the governing equations do not allow exact analytical solutions, so that either an analytical or a numerical approximation is required. Here we use a full-featured three-dimensional hydrodynamic circulation model developed by Blumberg and Mellor (1987). The model solves a finite difference approximation of the three-dimensional Navier-Stokes equations on a horizontal curvilinear grid with a terrain-following vertical σ coordinate ($\sigma = z/d$, where z is the dimensional vertical coordinate and d is depth). Vertical diffusivities for heat and momentum are calculated in the model using an imbedded second-order turbulence closure scheme (Mellor and Yamada 1974). The model uses a Smagorinsky-type formulation for horizontal diffusivity. This hydrodynamic model has been used extensively for simulating stratified flows in the coastal ocean, estuaries, and marginal seas.

To test the effect of stratification on mean circulation in a closed basin with a sloping bottom, the model was applied to a circular basin with a parabolic depth profile. The basin dimensions of 100-km diameter, 5-m minimum depth, and 100-m maximum depth were chosen to be typical of a large lake. The basin is assumed to be centered at 45°N. A rectangular horizontal grid with a uniform spacing of 2.5 km was used. In the vertical, 15 σ levels were used with closer spacing near the surface (layer thicknesses 0.011, 0.011, 0.023, 0.046, 0.091, 0.091, . . .). An initial temperature distribution of T = 15°C above 10 m, T = 5°C below 20 m, and a linear temperature gradient between 10 and 20 m was specified. Salinity was set as a constant 0.2‰ to be representative of freshwater. The model uses an equation of state where the density depends on salinity, temperature, and pressure (Mellor 1991). The model is started from an initial motionless state, and the free surface is initially flat. The initial vertical temperature profile described above is specified throughout the lake, and the model is allowed to adjust in time. A quadratic bottom friction law is used with a drag coefficient that depends on water depth and height of the lowest σ level above the bottom. The maximum drag coefficient for the circular basin was 0.33, the minimum 0.0054. The boundary conditions for temperature are no heat flux through the surface and no heat flux

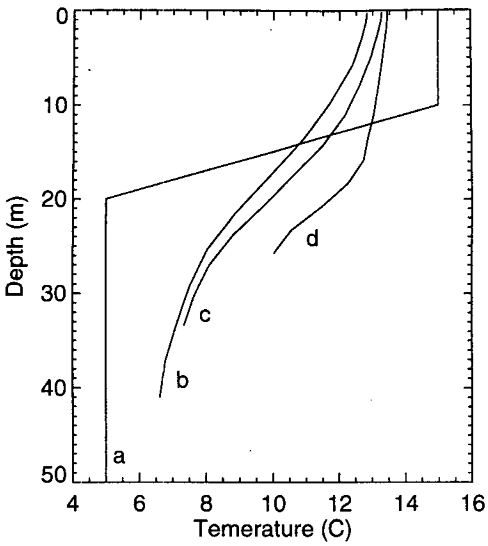

FIG. 2. Temperature profiles for initial condition (a) and final conditions (after 60 days) at points 11.25 km (b), 8.75 km (c), and 6.25 km (d) from shore in the circular basin.

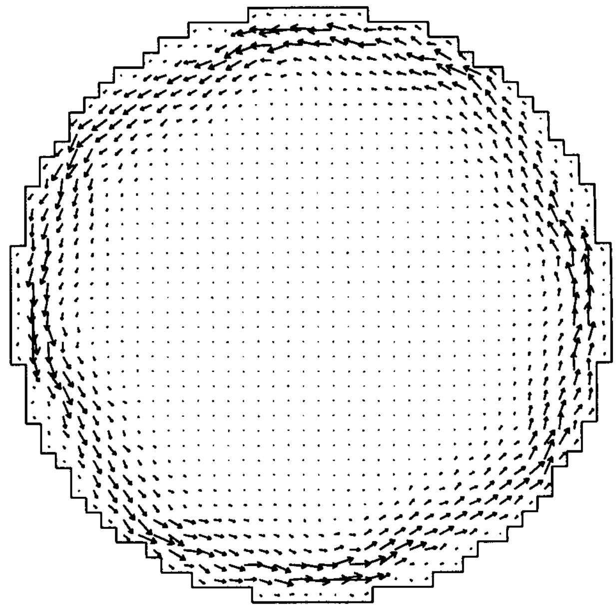

FIG. 3. Vertically averaged currents after 60 days in circular basin. The maximum velocity is 3.8 cm s^{-1}.

through the bottom. The lateral boundary condition is free slip.

3. Results

Figure 1 shows the initial and final temperature distributions in a vertical cross section through the center of the basin. After 60 days, the main features of the thermal structure and circulation pattern are fairly constant. Some thermal diffusion is still taking place, but the initial adjustment to the bottom boundary condition of zero heat flux has been accomplished mainly within the first 30 days. Diffusion has spread the vertical extent of the temperature gradient through the top 35 m. Near shore there appears to have been more vertical mixing than in the center of the basin. This is more clearly seen in Fig. 2, which depicts the original vertical temperature profile and the final profiles at three different points in the basin. Note that the nearshore temperatures are warmer than the offshore temperatures at all depths. This doming of the thermocline creates an internal pressure gradient in the region where the isotherms are tilted to satisfy the bottom boundary condition. There is a very small free surface response with a 0.4 cm decrease in elevation from the shore to the center of the basin. Because the higher pressure is at the shore, the internal pressure gradient is balanced by a cyclonic circulation around the basin. Figure 3 shows the vertically averaged velocity vectors in the numerical model after 60 days. The maximum speed is 3.8 cm s^{-1}. The surface current pattern after 60 days is almost identical, but the maximum speed is 7.9 cm s^{-1}. Figure 4 is a graph of the radial distribution of the vertically averaged azimuthal velocity component. The average azimuthal velocity is close to zero in the central region of the basin but increases to 1.3 cm s^{-1} between the 60-m isobath and the shore. We found that the magnitude of the vertically averaged velocity depends on the bottom drag coefficient, but is only reduced (or increased) by 20% for a corresponding increase (or reduction) of the bottom drag coefficient by a factor of 2.

The numerical model uses a second-order turbulence closure scheme to calculate vertical diffusivities. In the

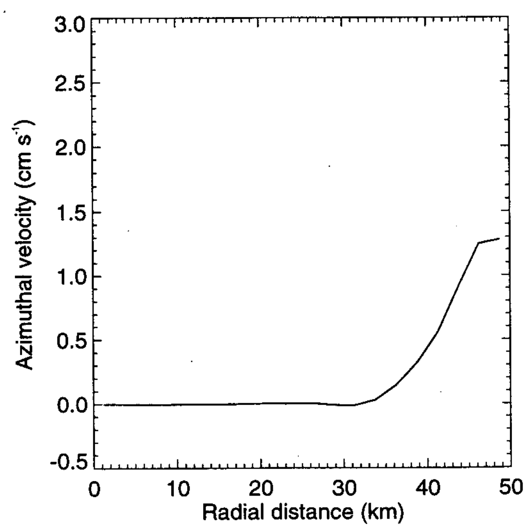

FIG. 4. Vertically averaged azimuthal velocity after 60 days in circular basin. Positive velocities are in the counterclockwise direction.

model simulation described above, the calculated values of the vertical diffusivities for heat and momentum are on the order of 10^{-5}–10^{-4} $m^2 s^{-1}$ in the thermocline region. The multiplier for the Smagorinsky horizontal diffusion term was chosen so that the average value of the horizontal diffusivity was 50 $m^2 s^{-1}$. Model simulations bypassing the turbulence closure calculations and using constant values for vertical heat and momentum diffusivities (3×10^{-5} and 2×10^{-5} $m^2 s^{-1}$, respectively) produced similar results to the variable vertical diffusivity case.

Additional model simulations were made to test the effect of increased horizontal and vertical resolution and horizontal diffusivity on the results. One was made with twice the horizontal grid resolution, 1.25 km instead of 2.5 km. Another used 27 vertical σ levels instead of 15. A third was made with no horizontal diffusivity. All three simulations produced similar results to the initial case. Based on the findings of Mellor et al. (1994), we did not expect the simulations with increased horizontal or vertical resolution to produce significantly different results from the base case. The main difference was a decrease in the magnitude of the azimuthal current near the shore in the 1.25-km grid case. We also found the magnitude of the cyclonic circulation increased with the value specified for the background vertical diffusivity in the model. If vertical diffusion is set to zero, no organized circulation pattern develops.

Two more model simulations were carried out with the 2.5-km grid to examine the effect of bottom slope on the results. In one simulation, depths were increased by a factor of 2 and in the other, decreased by a factor of 2. The results showed that the magnitude of the cyclonic circulation is inversely proportional to the bottom slope.

4. Discussion and conclusions

We have used a numerical hydrodynamic model with a boundary condition of no heat flux through the bottom to show that the equilibrium state of a stratified lake with a sloping bottom includes a cyclonic mean circulation. The characteristics of this response, namely, a dome-shaped thermocline and a mean cyclonic circulation, are commonly observed in large lakes. Current speeds in our model lake are comparable to observed mean circulations reported by Emery and Csanady (1973). The results are consistent with the classic observations of Harrington (1895) and Millar (1952) showing mean cyclonic circulation in the Great Lakes and cooler surface water temperatures in the deeper parts of the lakes, indicative of a domed thermocline. The results are also consistent with the dynamic height calculations of Ayers (1956) and Pickett and Richards (1975) for Lakes Huron and Ontario, respectively. In both studies, dynamic height is a minimum in the deeper parts of the basin, resulting in cyclonic flow. Endoh (1986) discusses topographic differential heating and wind stress curl as possible mechanisms for maintaining the persistent cyclonic flow observed in Lake Biwa during the stratified season. However, without the bottom boundary condition of zero heat flux, horizontal temperature gradients due to topographic differential heating would be dissipated by advection. The investigations of Beletskiy et al. (1991) of Lake Onega, Russia, also show persistent cyclonic circulation around a domed thermocline with a superimposed wind-induced circulation pattern.

The mechanism responsible for the cyclonic circulation reported in all these studies, namely, the doming of the thermocline in a lake with a sloping bottom does not depend on wind stress, wind stress curl, or asymmetry in the surface temperature field. The magnitude of the response depends directly on the density defect of the surface layer and inversely on the Coriolis force and the bottom slope. Our experiments with the numerical model indicate the maximum vertically averaged azimuthal velocity can be estimated as

$$v_{\max} = \frac{cgb}{fs},$$

where g is gravity, b is the density defect of the surface layer, f is the Coriolis parameter, s is the bottom slope, and c is a dimensionless constant, approximately 5×10^{-6} in our 2.5-km grid simulations. The results presented here will not be affected qualitatively by a small horizontally uniform geothermal heat flux through the bottom, as may be present in some lakes.

The results for the circular basin are part of the validation tests being performed on various numerical models to examine their ability to simulate long-term transport in the Great Lakes. Numerical circulation models must be used to simulate the long-term transport in large lakes and semienclosed marginal seas where complex physical processes are involved. For long-term studies, it is important to be sure that the numerical formulation correctly models the mean cyclonic circulation. Some three-dimensional circulation models do not include the physics necessary to model these processes. The Princeton model, with its terrain following σ coordinate, appears to be particularly useful for long-term simulations. For short-term studies, however, wind-induced circulations will often overwhelm the small residual circulation considered here and other types of hydrodynamic models may work just as well. We believe that the tendency of the thermocline to assume a dome-shaped configuration in long-term simulations is at least partly due to the bottom boundary condition of no heat flux. We do not propose that this mechanism is the only one responsible for observed mean circulation patterns, but we do feel that it is a significant contribution.

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