

Use of the Richards equation in land surface parameterizations

Deborah H. Lee¹

National Weather Service Ohio River Forecast Center, Wilmington, Ohio

Linda M. Abriola

Department of Civil and Environmental Engineering, University of Michigan, Ann Arbor

Abstract. Accurately modeling infiltration and soil moisture within land surface parameterization schemes (LSPs) of coupled land surface–atmosphere models is essential for producing realistic simulations of energy and moisture fluxes and for partitioning precipitation into infiltration, surface runoff, and drainage to groundwater. This report compares simulations of soil moisture, runoff, infiltration, and drainage to groundwater for a bare clay loam using three approaches: a finite difference solution of the vertically integrated Richards equation (an approach commonly used in LSPs), a highly resolved (spatially and temporally) finite element solution of Richards equation, and an analytical kinematic wave solution of Richards equation. Comparisons show that depth-averaged soil moisture simulated using the vertically integrated Richards equation is only similar to those of the finite element solution for vertical spatial discretizations finer than those employed by most state-of-the-art land surface–atmosphere transfer schemes. The vertically integrated Richards equation overpredicts soil moisture in the near-surface soil column and underpredicts drainage to groundwater. The infiltration formulation is found to be critical in partitioning precipitation into runoff, soil moisture, and drainage. Different infiltration formulations and vertical spatial discretizations may partly explain the very different land surface moisture and energy fluxes reported by the LSPs evaluated as part of the Project for Intercomparison of Land Surface Parameterization Schemes (PILPS) Phase 2(b) experiment.

1. Introduction

A number of studies [McCumber and Pielke, 1981; Pan and Mahrt, 1987] have shown that soil moisture is a fundamental determinant of land-to-atmosphere energy and moisture fluxes. Soil moisture limits soil water evaporation and vegetative transpiration and partitions latent and sensible heat fluxes to the lower atmosphere. Precipitation, infiltration, surface runoff, and drainage to groundwater (water leaving the unsaturated zone and entering the saturated zone) are also functions of soil moisture. Accurate representation of soil moisture within the land surface parameterization schemes (LSPs) of coupled land surface–atmosphere models is thus essential for producing realistic hydrometeorological simulations.

The Project for Intercomparison of Land Surface Parameterization Schemes (PILPS) Phase 2(b) [Shao and Henderson-Sellers, 1996] compared 15 LSPs using the high-quality, high-resolution HAPEX-MOBILHY data set of atmospheric fluxes, atmospheric forcing, soil moisture, hydrological fluxes, biomass accumulation, and surface properties (soil type, vegetation, and surface aerodynamic properties). The following conclusions of PILPS Phase 2(b) related the ability of LSPs to model soil moisture: (1) all schemes correctly described the annual cycle of soil moisture in a qualitative sense but produced profoundly different predictions of soil moisture with

the majority of schemes under predicting soil moisture for deep layers (below 0.5 m) while overpredicting soil moisture for the upper layers (above 0.5 m); (2) the partitioning of precipitation into evaporation and runoff plus drainage was very different in different schemes; (3) schemes exhibiting good soil moisture predictive capability had poor energy flux predictions and vice versa; (4) some schemes reproduced field measured soil moisture by simulation of large evaporative fluxes, while others had large runoff and drainage components; and (5) the partitioning of sensible heat and latent heat fluxes was closely related to that of runoff plus drainage and evaporation. Schemes with high runoff plus drainage resulted in low evaporation and high sensible heat flux, while schemes with low runoff plus drainage and high evaporation resulted in low sensible heat flux. Shao and Henderson-Sellers [1996] concluded that surface energy fluxes can never be correctly predicted unless runoff and drainage are correctly predicted. Even in simulations run with high-quality atmospheric forcing data and carefully chosen parameters, soil moisture prediction in climate change, weather forecast, or hydrological simulations cannot yet be considered reliable.

The PILPS 2(b) observation of profoundly different predictions of soil moisture is somewhat surprising since many state-of-the-art LSPs make use of the same nonlinear diffusion equation to describe transfer of water within the soil profile [McCumber and Pielke, 1981; Sellers, 1992; Shao and Henderson-Sellers, 1996]. The equation is a vertical integration over a soil layer of the Richards equation with the hydraulic conductivity, diffusivity, and matric potential related to the moisture content through a set of empirical relationships, generally those reported by Clapp and Hornberger [1978]. This vertically

¹Formerly at Great Lakes Environmental Research Laboratory, NOAA, Ann Arbor, Michigan.

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integrated Richards equation is used, along with a water-balance approach, to simulate the vertical distribution of soil moisture. Differences among soil moisture parameterization schemes are mainly related to the number of soil layers used (generally 1–8 layers), the thickness (1 cm to 1 m) of these layers, and the linkage between other LSP components (parameterizations for evapotranspiration, runoff, and drainage). Of the six multilayer diffusion type models tested in PILPS 2(b), one model had two soil layers, four models used three layers, and one model used five layers to represent a total soil depth of 1.6 m.

Well-known problems with state-of-the-art soil moisture parameterization schemes are that (1) they do not account for spatial variability of soil properties at model scales [Chen *et al.*, 1993]; (2) they lack data for both parameterization and validation [Liston *et al.*, 1994]; and (3) their equation nonlinearity makes sensitivity assessment difficult [Shao and Henderson-Sellers, 1996]. However, more fundamental problems may exist. As presently implemented in some LSPs, use of the vertically integrated Richards equation with coarse vertical discretization may not adequately define the soil moisture profile, especially near the surface. Additionally, parameterization of the boundary conditions could be improved. For example, infiltration parameterizations that are in part a function of the vertical discretization may not be appropriate owing to the coarseness of the discretization and other theoretical concerns. Boone and Wetzel [1996] demonstrated that the parameterization for land-atmosphere cloud exchange (PLACE) soil hydrology model (based on the vertically integrated Richards equation) is highly sensitive to vertical discretization and a lower boundary condition parameterization. Although it is well known in the soil science and hydrology literatures that the accuracy of solutions to the Richards equation are highly dependent on spatial discretization and parameterization of the boundary conditions, the implications of these factors on the predictions of atmospheric models seems underappreciated.

In this paper, the influence of vertical discretization and boundary condition parameterization (particularly infiltration) is explored through comparisons with the vertically integrated Richards equation approach as implemented in the Regional Atmospheric Modeling System (RAMS) [Pielke *et al.*, 1992] and to a highly resolved (spatially and temporally) finite element solution of the Richards equation [Abriola *et al.*, 1997]. This work differs from that of Boone and Wetzel [1996] in that they only intercompared low- and high-resolution simulations made with the same soil hydrology model. In addition to the highly resolved finite element solution of the Richards equation, we also make comparisons to a multiple precipitation event analytical kinematic wave approach for soil moisture modeling [Charbeneau, 1984]. The analytical kinematic wave approach (for a soil with vertically uniform properties) is not dependent on vertical spatial discretization, is without the instability and convergence problems associated with numerical solutions of the Richards equation, and is numerically efficient. It is included in the comparison to serve as an additional benchmark in evaluating the vertically integrated Richards equation approach and to explore its potential use as an alternative parameterization in LSPs. Soil moisture profiles and depth averages and soil column water balances are simulated using these approaches for a vertically uniform, bare clay loam subject to three discrete precipitation events within a 60-hour period. The effect of the vertical discretization of the integrated Richards equation model is evaluated through compar-

isons to the finite element solution and the analytical solution. We use temporal discretizations in keeping with those of atmospheric models and vertical discretizations required for numerical stability and convergence of the vertically integrated Richards equation formulation. This work also differs from that of Boone and Wetzel [1996] in that we focus on the effect of the parameterization of the upper boundary condition, whereas they explore the effect of the parameterization of the lower boundary condition. We investigate the effect of two different infiltration parameterizations (one of which is a function of the vertical soil discretization) on soil moisture and soil column water balances.

2. Integrated Richards Equation Model

Many state-of-the-art atmospheric LSPs make use of the vertically integrated Richards Equation for transfers of water within the soil profile. Richards equation, expressed in terms of θ , the volumetric soil moisture content ($\text{cm}^3 \text{cm}^{-3}$), is [Pielke *et al.*, 1992]

$$\frac{\partial \theta}{\partial t} = \frac{\partial W_s}{\partial z}, \quad (1)$$

where W_s , the moisture flux within the soil (cm s^{-1}), is defined as

$$W_s = D_\theta \frac{\partial \theta}{\partial z} + K_\theta. \quad (2)$$

Here D_θ is the moisture diffusivity ($\text{cm}^2 \text{s}^{-1}$) and K_θ is the soil hydraulic conductivity (cm s^{-1}). The variables K_θ and D_θ are typically related to θ through empirical relationships such as those of Clapp and Hornberger [1978] and Brooks and Corey [1964]. For the simulations presented here, the Brooks and Corey [1964] empirical functions are used:

$$D_\theta = \frac{K_{\theta_s}}{\alpha \lambda (\theta_s - \theta_r)} \left(\frac{\theta - \theta_r}{\theta_s - \theta_r} \right)^{2+1/\lambda} \quad (3)$$

$$K_\theta = K_{\theta_s} \left(\frac{\theta - \theta_r}{\theta_s - \theta_r} \right)^{3+2/\lambda}. \quad (4)$$

The values of θ_r (residual soil moisture content), θ_s (the saturated soil moisture content), λ , α , and K_{θ_s} (the saturated soil hydraulic conductivity) are empirically determined.

Vertically integrated over a soil layer k of thickness Δz , (1) becomes

$$\Delta z \frac{\partial \bar{\theta}}{\partial t} = W_{s_k} - W_{s_{k-1}}, \quad (5)$$

where $\bar{\theta}$ is the layer-average soil moisture. The RAMS numerically solves equation (5) using a block centered finite difference scheme in space and a forward finite difference scheme in time. The soil moisture $\bar{\theta}$ is calculated at the center of a soil layer, and the flux W_{s_k} entering and leaving the layer is calculated at the layer boundaries. The lower boundary value of $\bar{\theta}$ is set equal to a constant, θ_r , for the simulation duration. At each time step, $\bar{\theta}$ is computed in the next to deepest layer and the process proceeds to the surface layer. The equation is linearized by using the soil moisture content at time t to compute the hydraulic conductivity and diffusivity at time $t + 1$. Figure 1 illustrates the numerical scheme for a vertical discretization of six layers. The surface flux (infiltration) is calculated as the

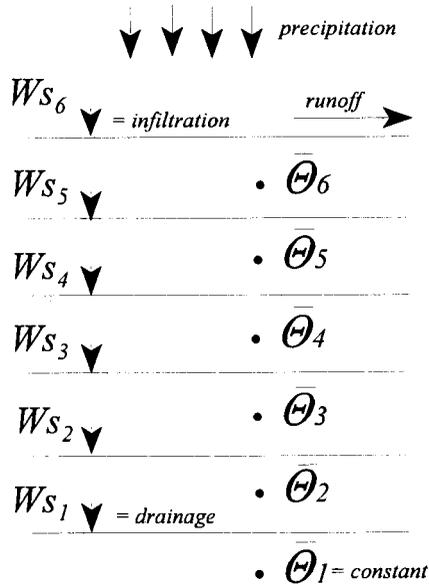


Figure 1. Integrated Richards equation numerical scheme implemented with six soil layers.

product of the precipitation rate, P , and the average available pore space in the top two layers:

$$W_{s_n} = P * \left[1 - \frac{(\bar{\theta}_n + \bar{\theta}_{n-1})}{2\theta_s} \right]. \quad (6)$$

The subscript n corresponds to the maximum number of soil layers; the top soil layer is assigned an index of n , and the deepest layer is assigned an index of 1 (see Figure 1).

Although this numerical scheme yields a stable solution, the formulation of the surface flux is problematic because it is indirectly a function of the layer thickness over which $\bar{\theta}$ is averaged. It neglects layer conductivity and does not allow infiltration to occur under saturated conditions (i.e., W_s in (6) is equal to zero when $\bar{\theta}_n$ and $\bar{\theta}_{n-1}$ are equal to θ_s). We also question the appropriateness of the lower boundary condition because it appears to be physically implausible. The lower boundary condition works numerically because computations proceed from the deepest layer to the surface.

At each simulation time step, the total soil moisture in the soil column is calculated by summing the product of $\bar{\theta}_k$ and Δz_k for k equal to 2 through n . The runoff rate is simply calculated as the difference between the precipitation rate and the infiltration rate. The drainage rate leaving the soil column is equal to the flux through the top of the lowest soil layer, calculated by the numerical solution of (5) (see Figure 1).

3. Multiple Precipitation Event Kinematic Wave Model

A multiple precipitation event analytical kinematic wave model is described by Charbeneau [1984]. This model neglects the influence of capillary spreading at the infiltration front; i.e., the diffusion term is dropped from the expression for the moisture flux in (2). The resulting quasi-linear partial differential equation is solved using the method of characteristics. Substitution of the derivative of the Brooks and Corey [1964] hydraulic conductivity relationship (equation (4)) into the characteristic equation yields analytical expressions that can be

solved for the soil moisture profile as a function of time and soil depth. During a precipitation event the soil moisture profile is modeled as a sharp front. After a precipitation event the drainage wave dominates the soil moisture profile. The rate of infiltration is equal to the hydraulic conductivity when the precipitation rate is less than the saturated hydraulic conductivity. When the precipitation rate is equal to or greater than the saturated hydraulic conductivity, the rate of infiltration is equal to the saturated hydraulic conductivity. The reader is referred to Charbeneau [1984] for complete details of the analytical solution.

At each time step of a simulation the total moisture in the soil column is calculated analytically by piecewise integration over each segment of the profile. As with the vertically integrated Richards equation model, the runoff rate is calculated as the difference between the precipitation rate and the infiltration rate. Drainage from the soil column is calculated as a residual of a water balance performed on the soil column at time t :

$$\begin{aligned} \text{drainage}_t = & \text{precipitation}_t - \text{runoff}_t \\ & - \Delta \text{total soil moisture}_{t-1 \rightarrow t}. \end{aligned} \quad (7)$$

Our implementation of Charbeneau's multiple event kinetic wave model was verified by reproducing the simulations reported by Charbeneau [1984].

4. Finite Element Richards Equation Model

The finite element solution of the Richards equation (pressure based formulation) was obtained using the simulator described by Abriola *et al.* [1997]. This model is a two-dimensional multiphase flow and compositional transport model originally designed to simulate bioventing systems. Governing equations are solved with a standard Galerkin finite element method using linear triangular elements. This model allows boundary conditions to be specified as a constant head or flux. The model has been tested and verified as reported by Abriola *et al.* [1997] for both single phase and multiphase flow.

The total soil moisture in the soil column is calculated by numerical integration of the soil moisture profile. As with the models described above, the runoff rate is calculated as the difference between the precipitation rate and the infiltration rate. Drainage from the soil column is calculated as a residual of a water balance (equation (7)).

5. Comparison of Models

Two comparisons are made below to illustrate model performance. The first comparison highlights the importance of adequate vertical discretization (soil layer thickness) in simulating soil moisture profiles and water balances with the integrated Richards equation model. For this comparison, the surface moisture flux (i.e., the upper boundary condition) is the same for all three models and is equal to the precipitation rate. The second comparison illustrates the importance of the infiltration formulation in partitioning the precipitation into runoff, infiltration, soil moisture, and drainage. For this comparison, the models use their respective infiltration formulations as described in the previous sections. For each comparison, one simulation is made with the analytical and finite element models (identical in both sets) and two are made with the integrated Richards equation model. The simulations with the

Table 1. Brooks and Corey Soil Water Retention and Hydraulic Conductivity Parameters

Parameter	Value
K_s	4.167 cm h ⁻¹
θ_s	0.520 cm ³ cm ⁻³
θ_r	0.246 cm ³ cm ⁻³
ε	4.25
λ	1.60
α	0.08 cm ⁻¹

integrated Richards equation model differ in the number and thickness of the soil layers but maintain the same total soil depth.

5.1. Initial and Boundary Conditions

Identical precipitation sequences and soil properties are input to all simulations. The 60-hour simulation period has three precipitation events. The first begins immediately with precipitation of 2 cm h⁻¹ for 6 hours, the second begins at 24 hours with precipitation of 1 cm h⁻¹ for 6 hours, and the third begins at 44 hours with precipitation of 1 cm h⁻¹ for 4 hours. A bare, homogenous clay loam soil of 150-cm depth is specified. The empirical soil water retention and hydraulic conductivity values summarized in Table 1 are taken from Charbeneau [1984]. An initial uniform soil moisture content profile of 0.246 cm³ cm⁻³ (the residual soil moisture value from Table 1) is assumed. Note that these conditions are identical to those used by Charbeneau [1984] with the exception of the longer simulation period and the third precipitation event. Soil moisture profiles, total column soil moisture, and cumulative runoff, infiltration, and drainage are reported at intervals of 15 min for all simulations.

For the integrated Richards equation model the soil column is divided into six uniform layers, each 30 cm thick, and then alternatively, into 51 uniform layers, each 3 cm thick (referred to as IRE-A and IRE-B, respectively, in the following text and figures). Because the deepest layer is assigned a constant soil moisture content, it is excluded from the total soil moisture computations (only the top 150 cm are considered). The time step of the simulation with the coarser vertical discretization is 15 min and that of the finer vertical discretization is 90 s, yielding stable numerical solutions. The lower boundary soil moisture condition for the model is held at a constant 0.246 cm³ cm⁻³ throughout the simulation.

In the Richards equation model the soil column is divided into layers 0.5 cm thick throughout a total depth of 2.5 m; however, only the top 150 cm are considered in the total column soil moisture and water balance computations. The deeper soil column is used with this model to ensure a physically plausible treatment of the lower boundary condition. The lower boundary condition is specified at saturation to simulate the capillary fringe. The upper boundary condition is a specified moisture flux, equivalent to the precipitation rate. The time step is variable, ranging between ~0.1 and 1 s. The fine temporal and vertical discretization is used here to provide the best possible standard for comparison to the other models' results.

5.2. Effect of the Vertical Soil Discretization

Figure 2 compares the soil moisture profiles simulated with the models for selected times. The profiles of the integrated

Richards equation model with the 3-cm discretization (IRE-B) closely approximate those of the finite element Richards equation model, except at the leading fronts of the drainage waves. The analytical solution also closely approximates the profiles simulated with the finite element Richards equation model except at the near surface following a precipitation event and at the leading fronts of the drainage waves. The IRE-A profiles (30-cm discretization) have notable differences throughout the soil depth and simulation period. For example, at time $t = 7$ hours (Figure 2) the profile computed by the IRE-A model overpredicts the surface soil moisture in comparison to the finite element Richards equation model by 27%, while the same model with finer vertical discretization (IRE-B) overpredicts by only 3%. The analytical solution underpredicts the surface soil moisture by 31%. Throughout the simulation period, differences in the surface soil moisture range from -37% to 7% for the analytical model, -41% to 27% for the IRE-A simulation, and -11% to 4% IRE-B simulation. The inaccurate near-surface soil moisture prediction of the analytical solution makes it a poor candidate for exclusive use in LSPs but may have potential for simulating deep soil moisture when coupled with a high-resolution numerical solution of the Richards equation at the near surface. The integrated Richards equation solution can be improved by increasing the vertical resolution (as expected), but even the 3-cm resolution still yields substantial differences from the finite element solution of Richards equation.

Because soil moisture in the near-surface soil column is an important determinant of moisture and energy fluxes to the atmosphere, comparison of depth-averaged soil moistures is of interest. Table 2 summarizes the relative percent differences of depth-averaged soil moistures computed using the integrated Richards equation model and the analytical model to those computed using the finite element Richards equation model. For the first centimeter of the soil profile the relative percent differences range from -40% to 25% for the IRE-A simulation (30-cm discretization) and range from -9% to 2% for the IRE-B simulation (3-cm discretization). Likewise, the relative percent differences range from -24% to 9% for the analytical model. Because all schemes are mass conservative, the relative differences tend to decrease with an increasing soil depth over which the average is taken. The relative percent differences become small (2% or less) for depths greater than 30 cm except for the IRE-A.

Table 3 summarizes the water balance components (cumulative precipitation, runoff, infiltration, drainage, and change in total column soil moisture) at the end of the simulation. The partitioning of the precipitation is not very different between the models due to the same precipitation input and infiltration flux. There is no runoff throughout the simulation period (the precipitation rates are less than the saturated hydraulic conductivity: -4.167 cm h⁻¹ versus the maximum precipitation rate of 2 cm h⁻¹). Drainage begins at 43 hours for the Richards equation model, at 44 hours for the analytical and IRE-B simulations, and at 32 hours for the IRE-A simulation. Despite the earlier occurrence of drainage with the IRE-A simulation the cumulative drainage is less than that of the Richards equation simulation (2.59 versus 3.22 cm). The differences in the change in total soil moisture reflect the differences in drainage, with the IRE-A simulation retaining the most soil moisture at the end of the simulation period.

Comparison of these simulations reveals that the integrated Richards equation model with coarse vertical discretization

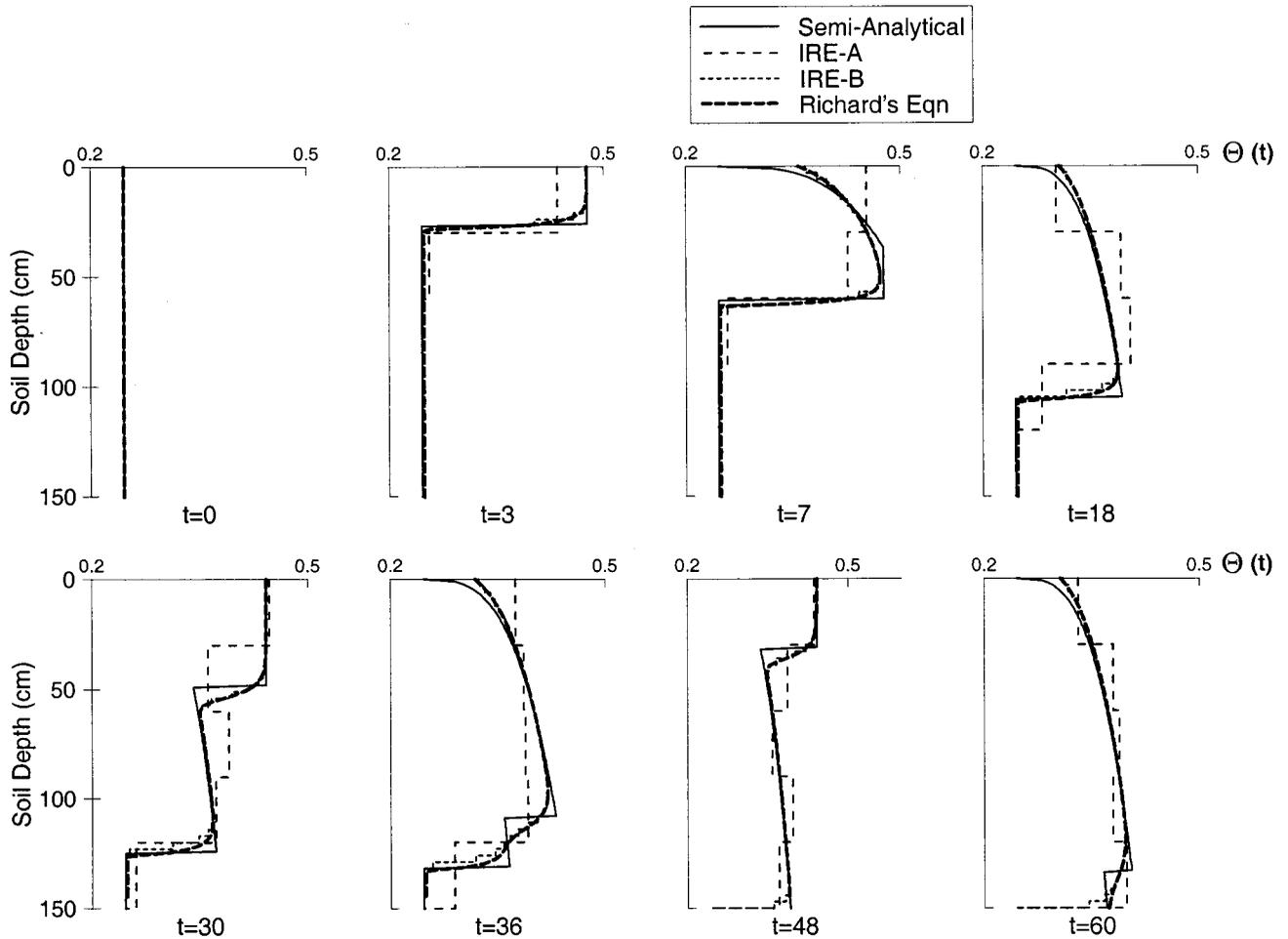


Figure 2. Comparison of the soil moisture profiles calculated with the analytical and integrated Richards equation models with the same infiltration parameterization.

predominantly overestimates near-surface soil moisture. Because accurate estimates of near-surface soil moisture are important to correctly estimate soil water evaporation and shallow-root vegetation transpiration, the use of the integrated Richards equation model with coarse vertical discretizations biases computations of moisture and energy flux calculations in LSPs. As noted by *Boone and Wetzel* [1996], the soil water profile has a large effect on the partitioning of surface energy between sensible and latent heat fluxes, resulting in a large effect on the numerical simulation of the parent atmospheric

model, at both climate and regional scales. Use of the integrated Richards equation with coarse vertical discretization in land surface-atmosphere transfer schemes may in part explain the PILPS 2(b) finding that the majority of schemes underpredict soil moisture for deep layers while overpredicting soil moisture for the upper layers. A deeper analysis of the PILPS 2(b) results cannot be made here because the detailed differences among the large number of complicated nonlinear schemes tested make it impossible to determine the effect of these differences on the overall performance [*Shao and Hen-*

Table 2. Depth-Averaged Soil Moisture Maximum, Minimum and Average Percent Differences of the Integrated Richards Equation Simulations and the Analytical Solution Relative to the Finite Element Richards Equation Simulation (IRE-A: 30-cm Resolution, IRE-B: 3-cm Resolution)

Soil Depth, cm	Analytical			IRE-A			IRE-B		
	Maximum, %	Minimum, %	Average, %	Maximum, %	Minimum, %	Average, %	Maximum, %	Minimum, %	Average, %
0-1	9	-24	-12	25	-40	5	2	-9	1
0-3	6	-15	-8	23	-38	4	0	-1	0
0-6	5	-10	-6	20	-35	3	0	-1	0
0-15	-1	-9	-7	11	-25	-2	-3	-4	-3
0-30	1	-4	-3	9	-10	0	0	-1	0
0-60	0	-2	-2	6	-6	1	0	-1	0
60-150	1	-2	0	4	-7	-1	0	-2	-1

Table 3. Water Balance Components at the End of the Simulation Period

Simulation	Cumulative Precipitation, cm	Cumulative Runoff, cm	Cumulative Infiltration, cm	Change in Total Soil Moisture, cm	Cumulative Drainage, cm
<i>Simulation Results With Infiltration as a Function of the Surface Hydraulic Conductivity</i>					
Richards equation	22.00	0.00	22.00	18.78	3.22
Analytical	22.00	0.00	22.00	18.30	3.70
IRE-A	22.00	0.00	22.00	19.40	2.59
IRE-B	22.00	0.00	22.00	18.64	3.36
<i>Simulation Results With Infiltration as a Function of the Average Available Near-Surface Pore Space</i>					
IRE-A	22.00	14.95	7.05	6.98	0.07
IRE-B	22.00	17.51	4.49	4.42	0.07

erson-Sellers, 1996]. However, the similarity of findings is noteworthy.

5.3. Effect of the Infiltration Formulation

Figure 3 compares the soil moisture profiles of the Richards equation model to those of the integrated Richards equation model with an infiltration parameterization as a function of the near-surface average available pore space (equation (6)). The analytical model results are also shown. The different infiltration parameterization results in very different infiltration rates and the IRE profiles differ significantly from the finite element Richards equation model profiles. Differences in infiltration

predictions also occur between the two integrated Richards equation model simulations despite use of the same infiltration parameterization. Infiltration predictions are different because the infiltration formulation is a function of the vertical soil discretization. Less water infiltrates in the simulation with the finer vertical discretization and more infiltrates for the simulation with the coarser vertical discretization.

The IRE-A and IRE-B water balance components reflecting the change in the infiltration formulation are shown at the bottom of Table 3. Although the precipitation input is the same for all of the simulations, the precipitation partitioning due to the infiltration parameterization is significantly differ-

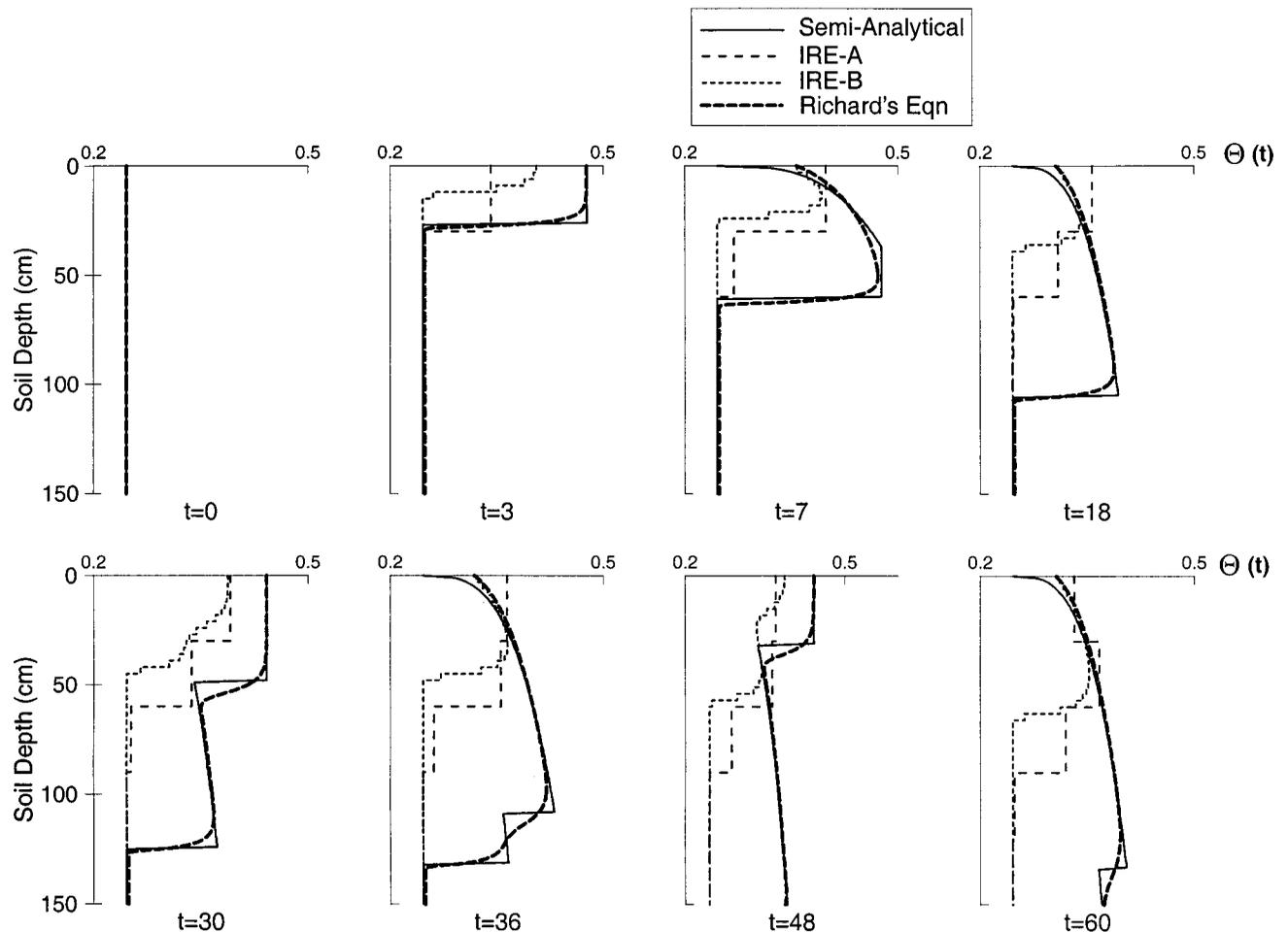


Figure 3. Comparison of the soil moisture profiles calculated with the analytical and integrated Richards equation model with different infiltration parameterizations.

ent. In comparison to the simulations with the infiltration as a function of the hydraulic conductivity, the cumulative infiltration of the latter simulations decreases from 100% of the cumulative precipitation to 32% (IRE-A) and 20% (IRE-B). The runoff increases from 0% of the precipitation to 68% (IRE-A) and 80% (IRE-B). The cumulative drainage decreases from 12% to 15% of the precipitation to less than 1%. The change in total soil moisture also decreases from 83% to 88% of the cumulative precipitation to 32% (IRE-A) and 20% (IRE-B).

As these results show, the partitioning of precipitation into runoff, infiltration, soil moisture, and drainage is largely determined by the infiltration parameterization. An infiltration parameterization that is a function of vertical soil discretization will result in different infiltration rates, and hence precipitation partitioning, for simulations that are identical in all ways except the vertical discretization. A simple formulation that is a function of the hydraulic conductivity but independent of the vertical resolution is recommended until such time as the spatial (horizontal and vertical) resolution of LSPs justify a more theoretical approach. One such formulation is that used by Charbeneau [1984] when $P < K_{\text{sat}}$, $W_s = P$ and when $P \geq K_{\text{sat}}$, $W_s = K_{\text{sat}}$.

Different infiltration parameterizations may partially explain the PILPS 2(b) finding that the partitioning of precipitation was very different in different schemes. Because the partitioning of sensible heat and latent heat fluxes is closely related to the precipitation partitioning, different infiltration formulations may also explain why the PILPS 2(b) experiment models had very different partitioning of the surface energy fluxes. As noted above, a more definitive analysis of the PILPS 2(b) results cannot be made here, but the similarity in findings is again noteworthy.

6. Conclusions

Land surface-atmosphere transfer schemes are complex. The numerical experiments reported here are simplistic in that they only consider two parameterizations (infiltration and vertical soil moisture movement) of the many that comprise the schemes for simulating the moisture and energy fluxes between the earth's surface and the atmosphere. Despite the simplicity of these experiments, however, the simulations offer a basis for explaining some of the major findings of the PILPS 2(b) experiment. At a minimum the results point to parameterization refinements that could be readily implemented to improve land surface-atmosphere transfer schemes, and ultimately, hydro-meteorological simulations. Specifically, soil moisture profiles simulated with the integrated Richards equation model approach those of the Richards equation model only for fine vertical soil discretization, finer than appears to be used by many land surface-atmosphere transfer schemes as reported in the literature. Coarse vertical discretization results in large overestimates and underestimates of near-surface soil moisture with overestimates predominating. Coarse vertical discretization may partially explain the over prediction of upper layer soil moisture reported for most of the PILPS 2(b) schemes.

Additionally, the treatment of the upper and lower boundary conditions should be revisited. The infiltration formulation is key to determining the partitioning of the precipitation into runoff, infiltration, soil moisture, and drainage. Infiltration parameterizations that depend upon the available pore space in

the upper soil layers result in infiltration predictions that are a function of the vertical soil discretization. This type of parameterization is undesirable because it does not permit infiltration when upper soil layers are saturated and infiltration is substantially less than that predicted by a simple but more theoretically sound parameterization. Different infiltration formulations may partly explain why the PILPS experiment models have very different partitioning of precipitation. Because the partitioning of sensible heat and latent heat fluxes is closely related to the precipitation partitioning, different infiltration formulations may also explain why the PILPS experiment models had very different partitioning of the surface energy fluxes. Correct treatment of the lower boundary condition is also necessary; Boone and Wetzel [1996] have shown it is important for accurately modeling groundwater recharge and deep soil moisture.

Comparison of the analytical soil moisture model to the Richards equation model provides insights into the analytical model's ability to adequately model vertical soil water movement. The limitations of the analytical model, primarily its neglect of capillary drive and poor estimation of near-surface soil moisture, the difficulty of incorporating soil moisture losses due to other processes such as plant transpiration, and the requirement that precipitation be modeled as discrete stepwise pulses may preclude its use in coupled land surface-atmosphere models for simulating near-surface soil moisture. It may have utility in LSPs for simulating deep soil moisture when coupled with a high-resolution numerical solution of the Richards equation for the near surface. Equivalent improvements may occur by using the integrated Richards equation model with fine vertical discretizations and improved infiltration formulations.

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- L. M. Abriola, Department of Civil and Environmental Engineering, Suite 181 EWRE Building, University of Michigan, Ann Arbor, MI 48109-2125. (Linda_Aabriola@um.cc.umich.edu)
- D. H. Lee, NWS Ohio River Forecast Center, 1901 S. State Street, Route 134, Wilmington, OH 45177. (deborah.lee@noaa.gov)

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